Sparse sampling in MRI:

From basic theory to clinical application

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Objective

- Provide an intuitive overview of compressed sensing as applied to MRI

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- Provide an intuitive overview of compressed sensing as applied to MRI
- This is not a research talk
  - I will use my data to illustrate points
- This is an emerging technology
  - There are unanswered questions
  - Some of this talk will be speculative

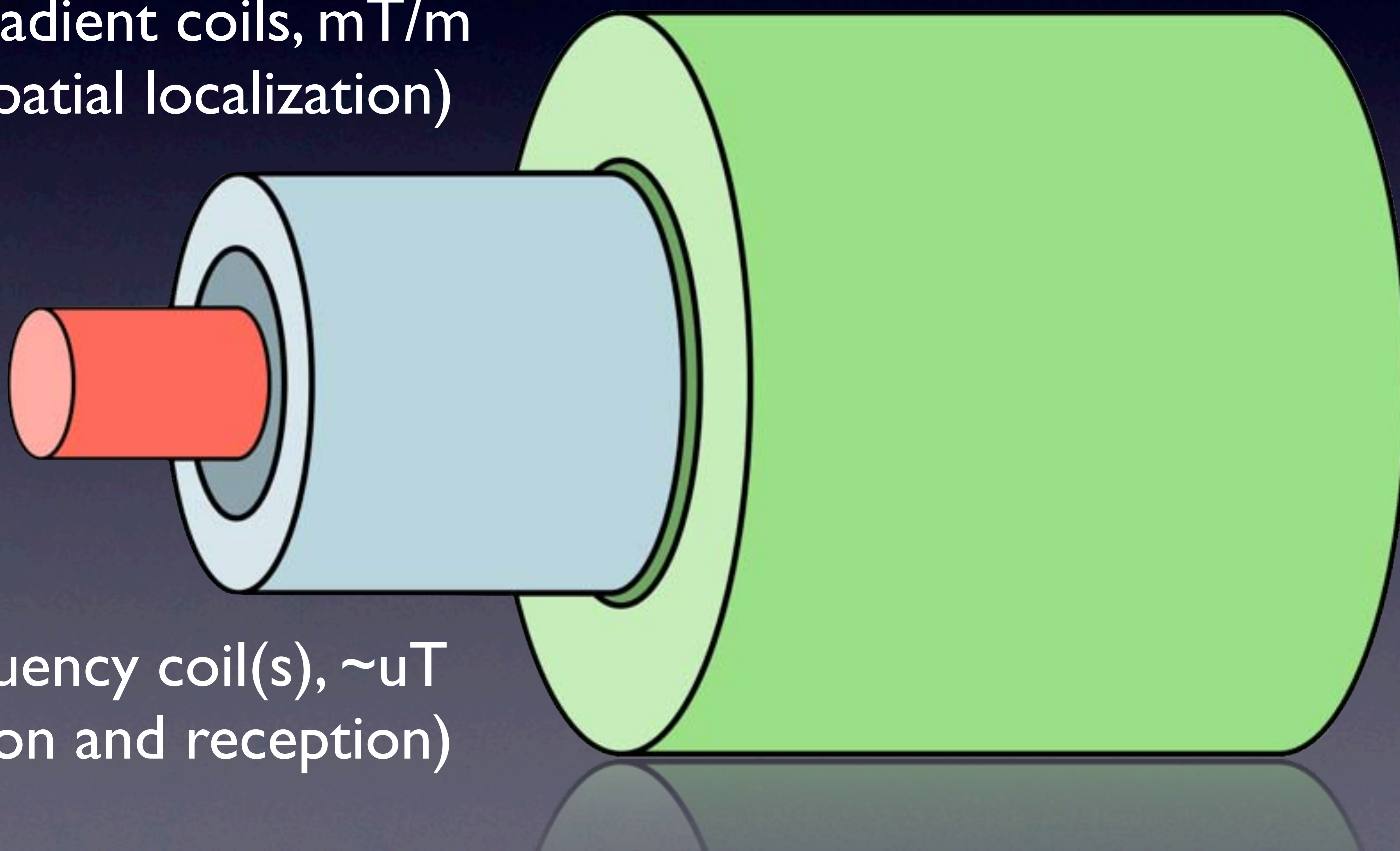
Outline

- MR Physics
  - k-space and sampling requirements
- Compressed sensing
  - Constrained reconstruction, sparsity, and random sampling
- Applications
  - Neuroimaging

Magnetic Resonance Imaging

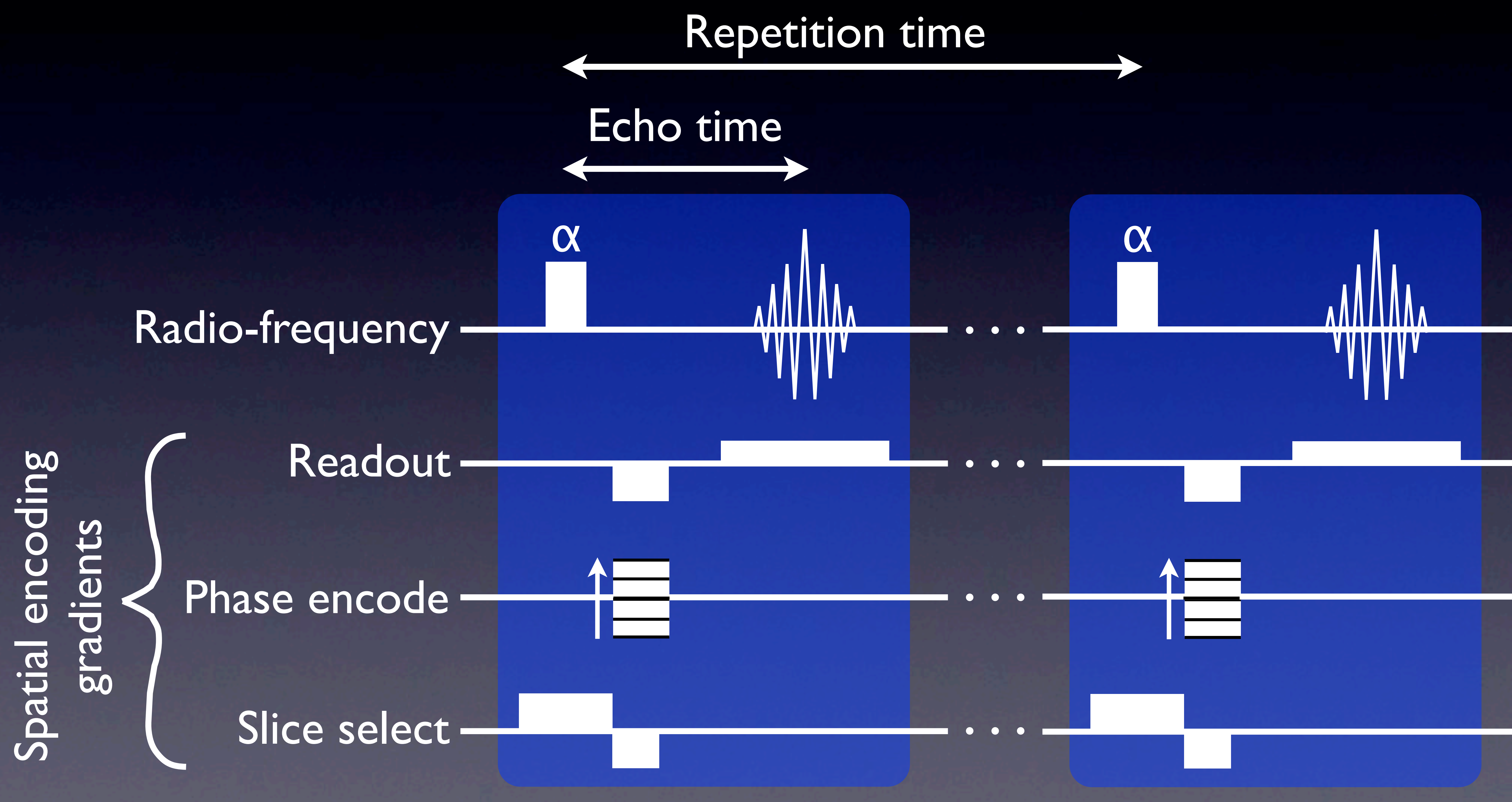
Main magnet, ~1 Tesla
(magnetize the sample)

Gradient coils, mT/m
(spatial localization)



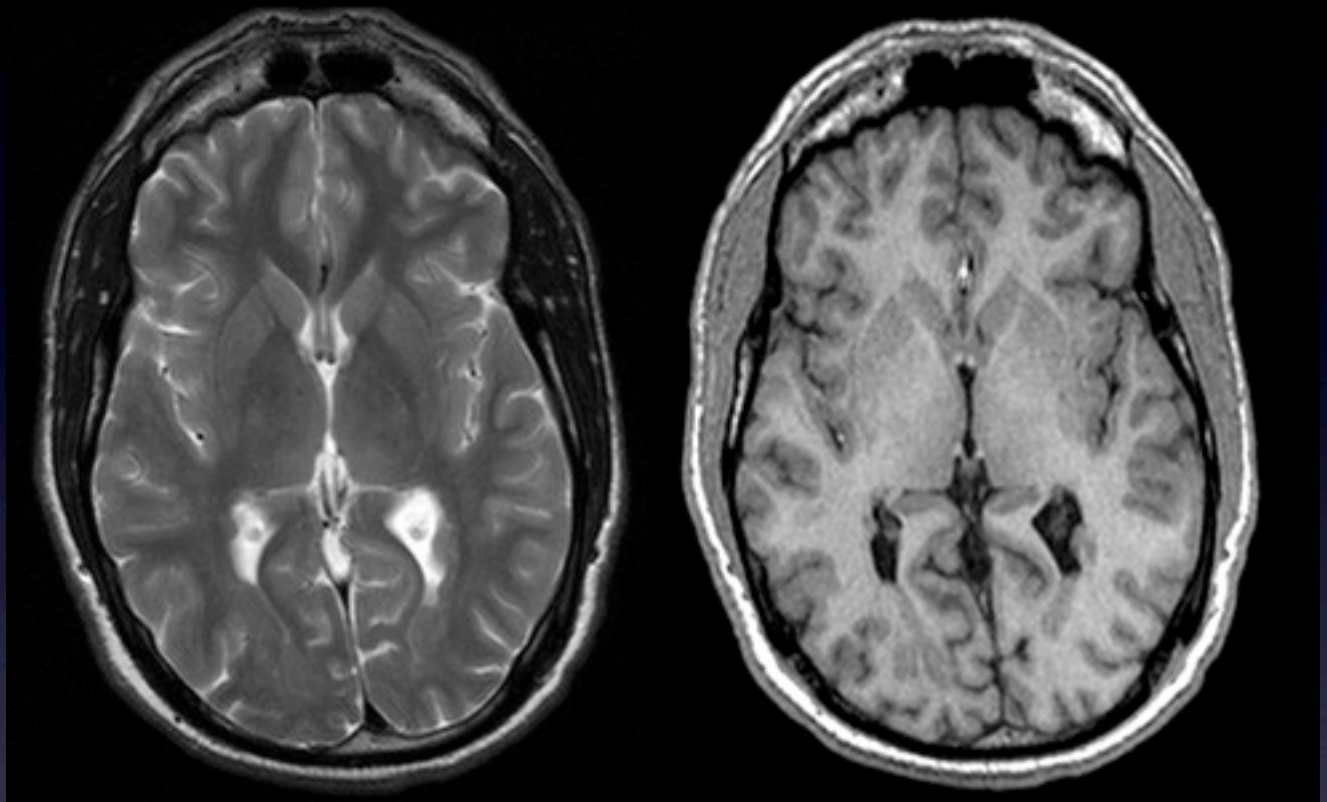
Radio-frequency coil(s), ~uT
(transmission and reception)

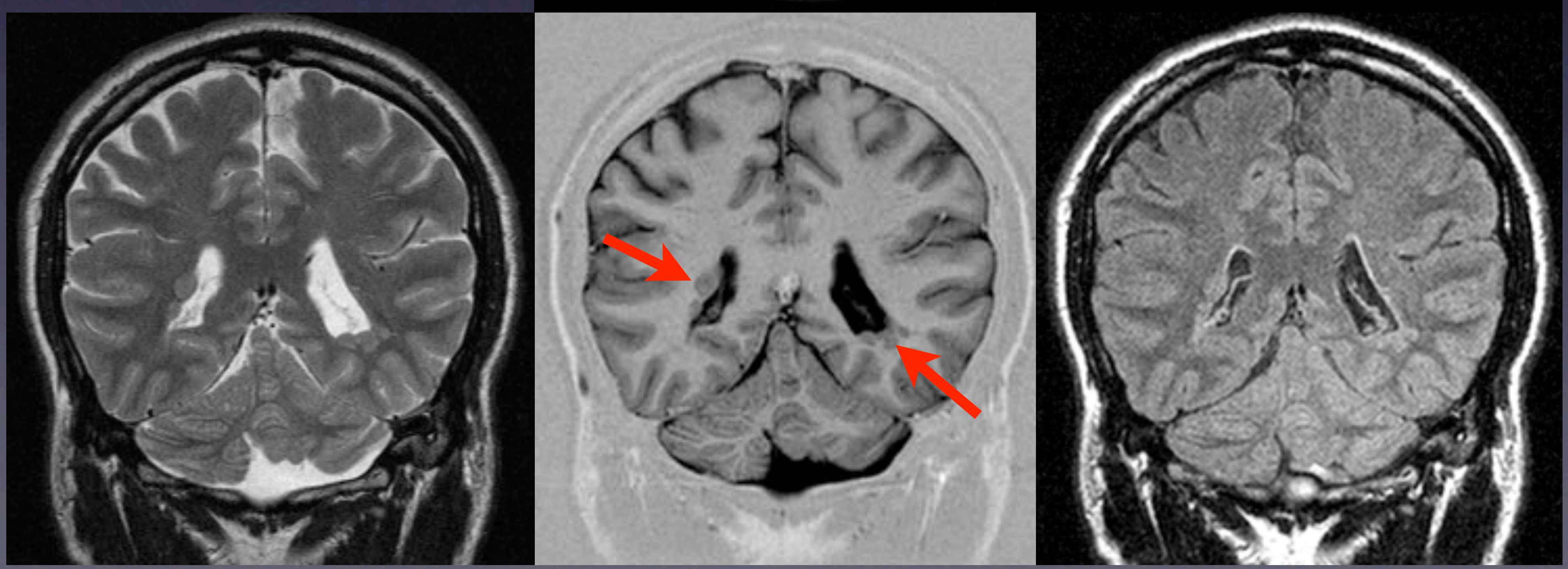
Magnetic Resonance Imaging



Magnetic Resonance Imaging

- Non invasive
- High resolution
- Multiple intrinsic contrast mechanisms
- Arbitrary slice orientation





Magnetic Resonance Imaging

Acquisition time

Spatial encoding

- Serially acquire all of the points in an image
- Solutions
  - Adjust resolution and field-of-view to require fewer points
  - Undersample the data

Low sensitivity

- Very few spins contribute signal; lots contribute noise
- Solutions
  - Adjust resolution
  - Scan longer

Magnetic Resonance Imaging

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Undersample without noise amplification

Magnetic Resonance Imaging

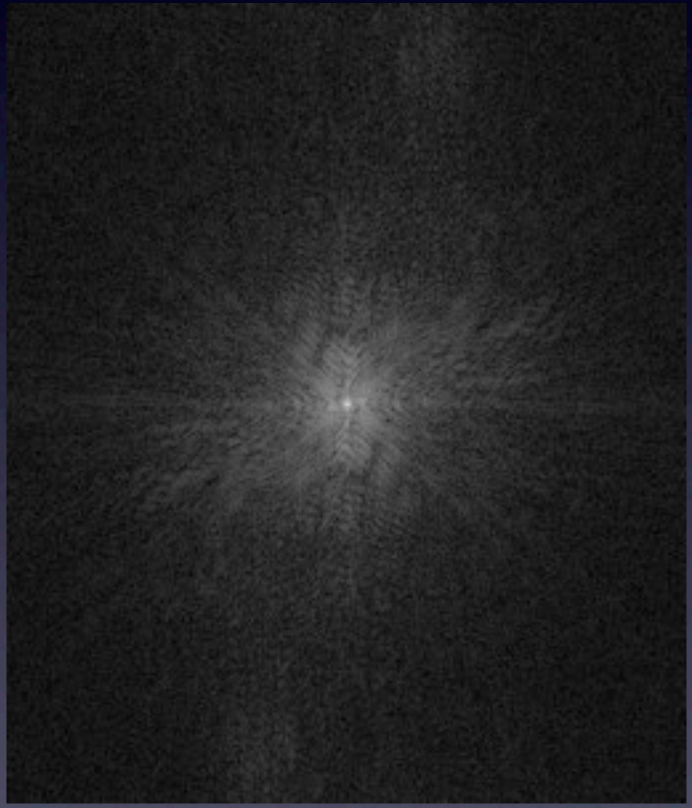
Sampling requirements

k-space*

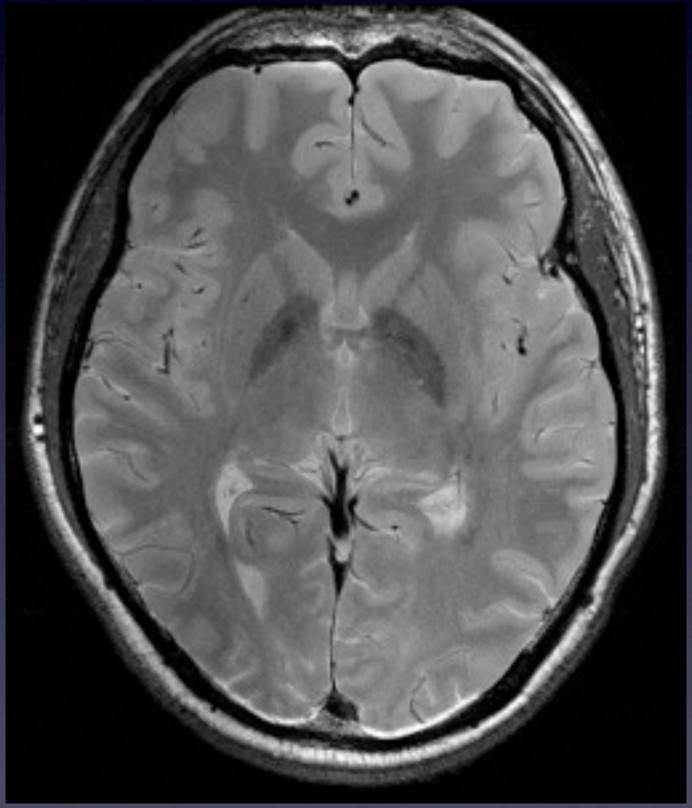
image-space

dk_y

FOV_{ky}



Fourier Transform



dy

FOV_y

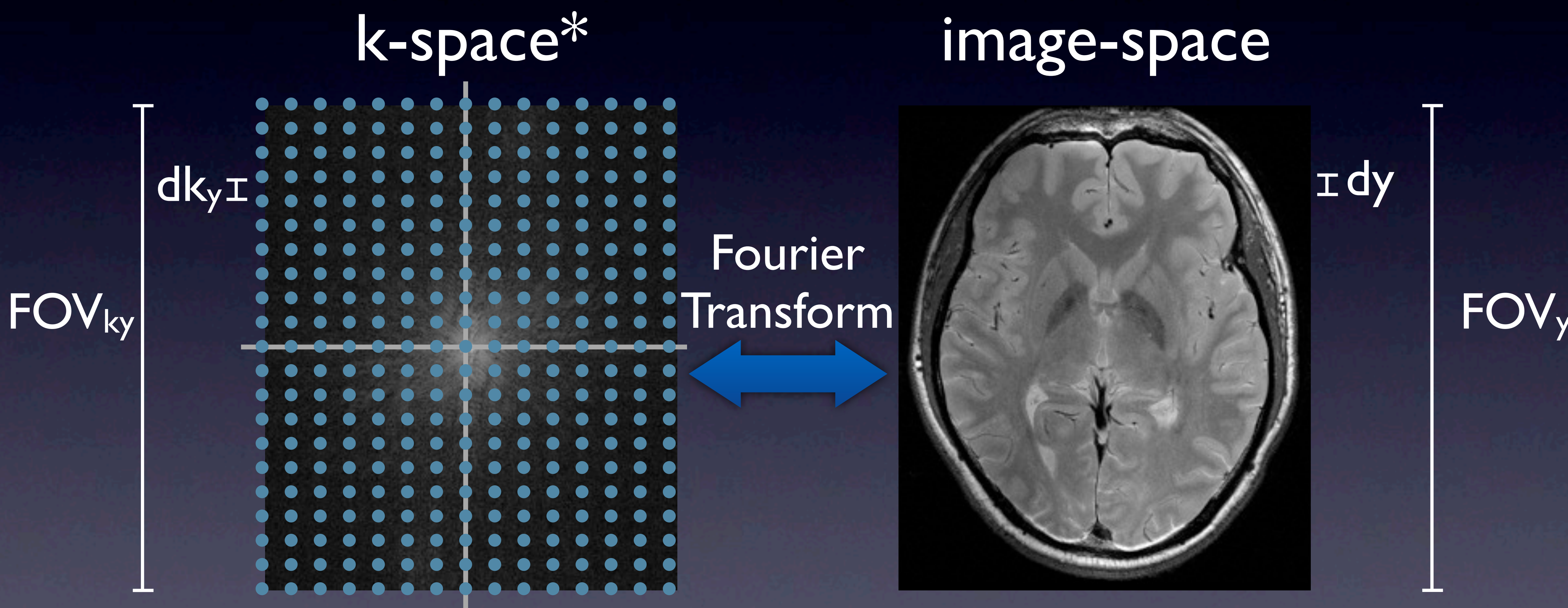
$FOV_y = 1/dk_y$

$dy = 1/FOV_{ky}$

*All k-space images are logarithmically scaled

Magnetic Resonance Imaging

Sampling requirements



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Magnetic Resonance Imaging

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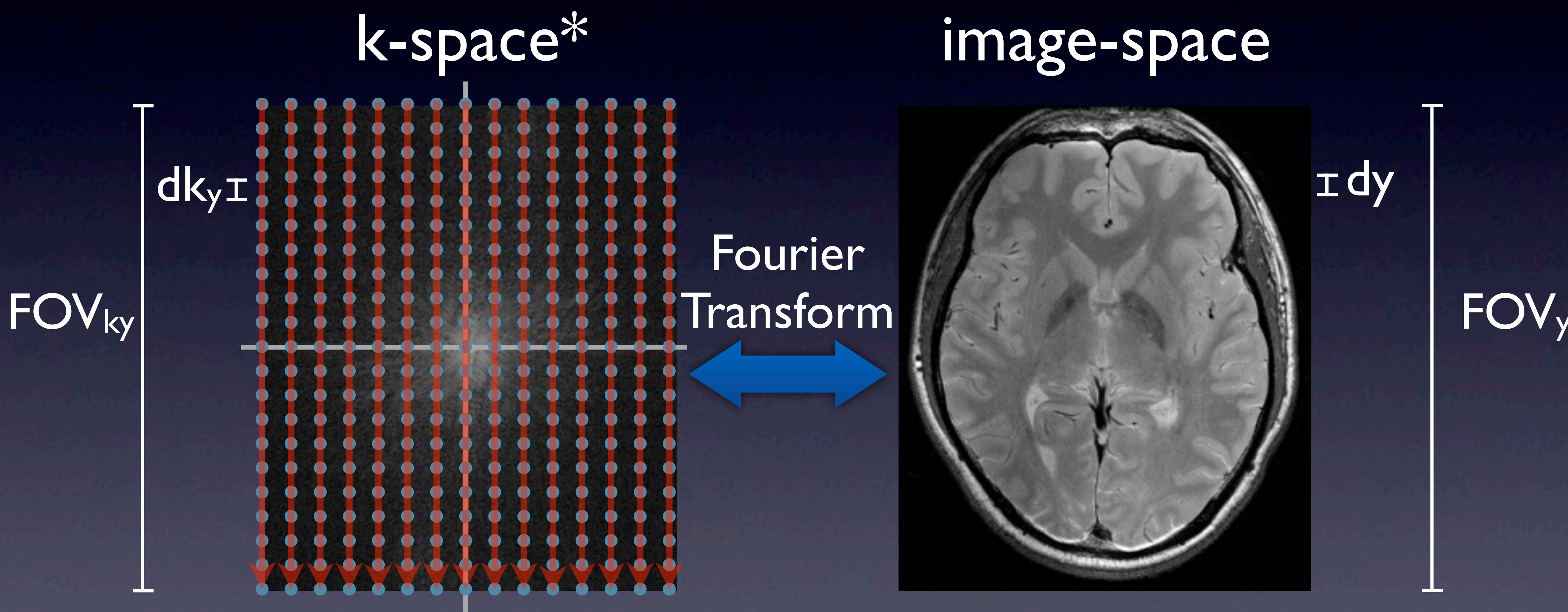

$FOV_y = 1/dk_y$

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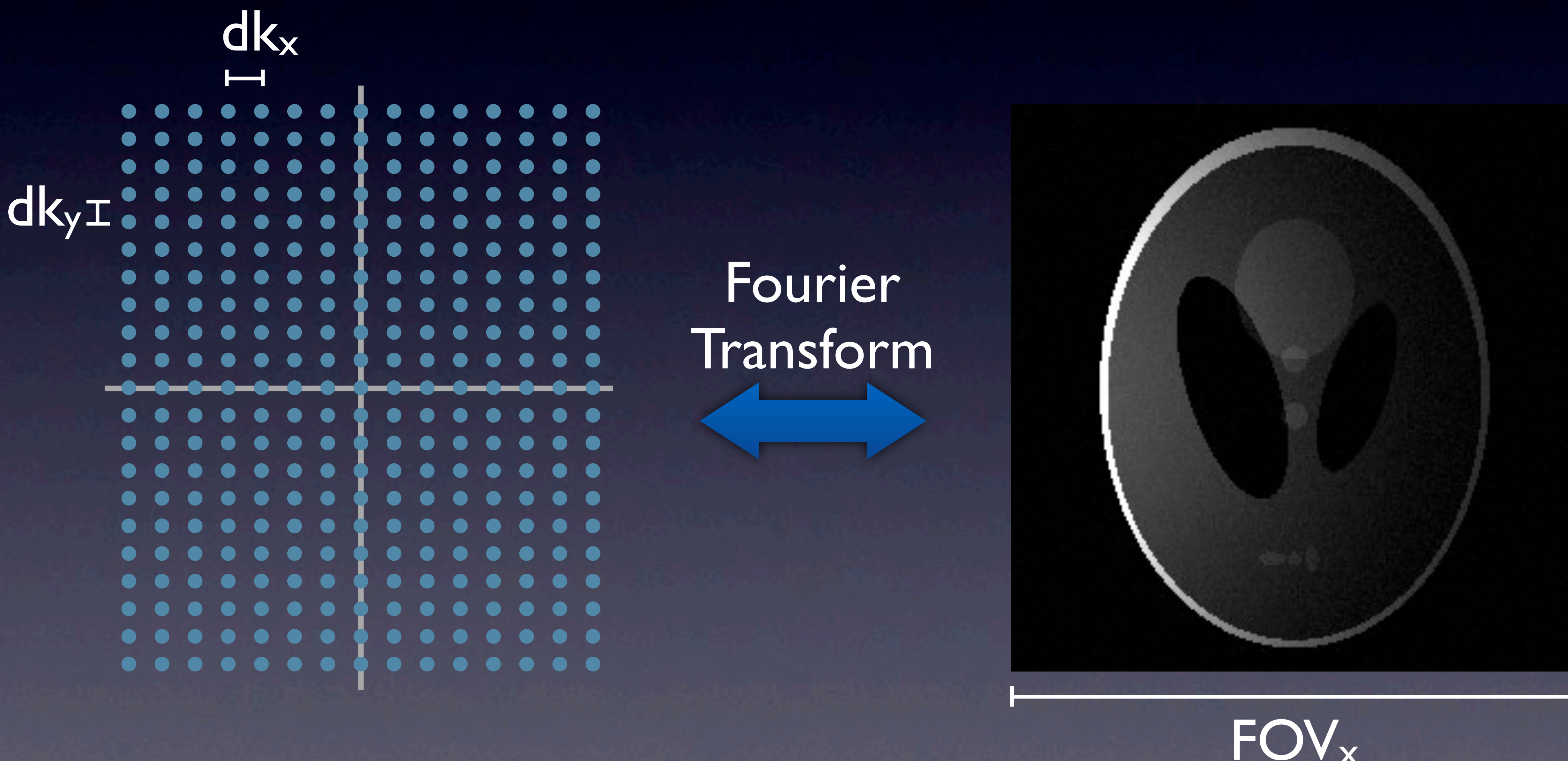
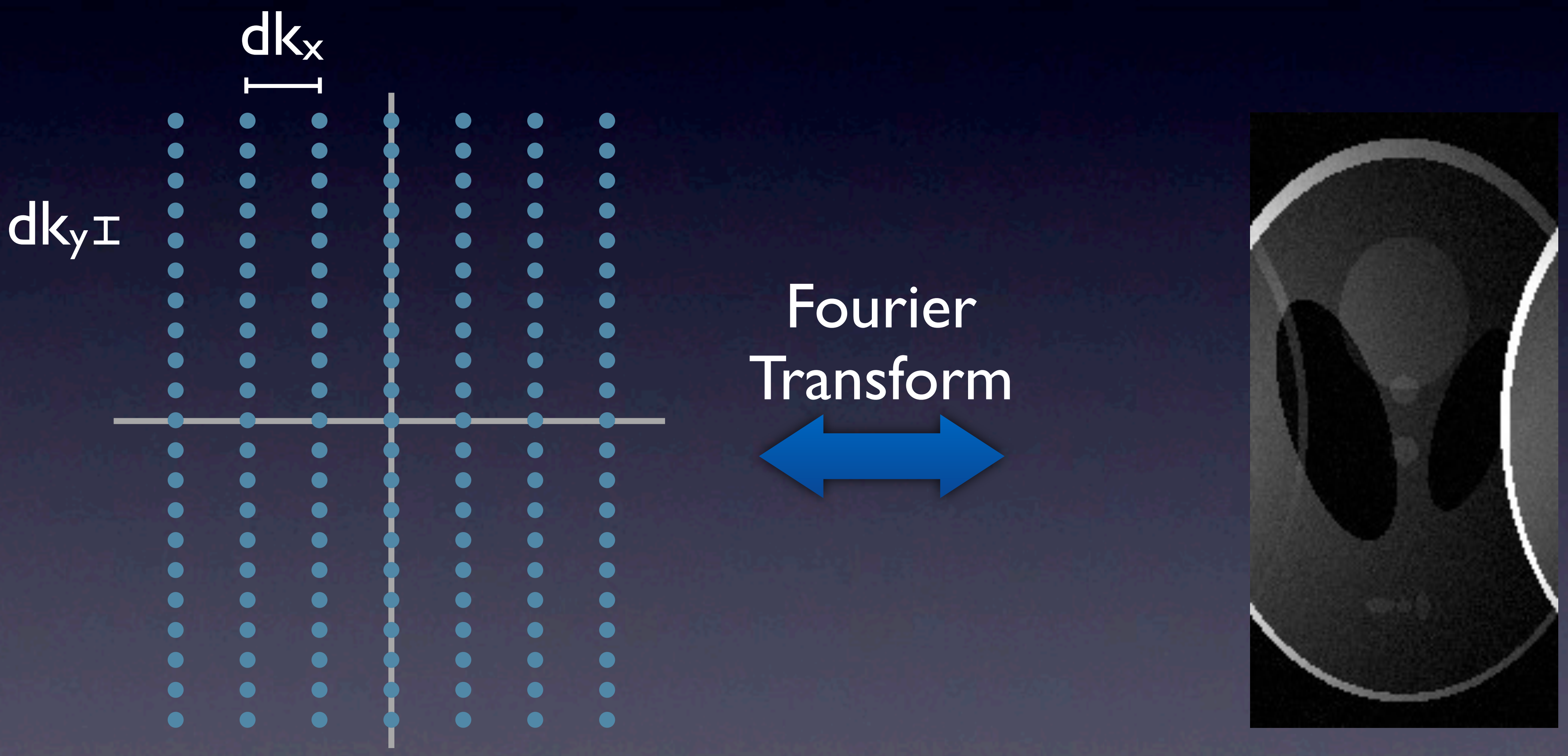
Magnetic Resonance Imaging

Undersampling



Magnetic Resonance Imaging

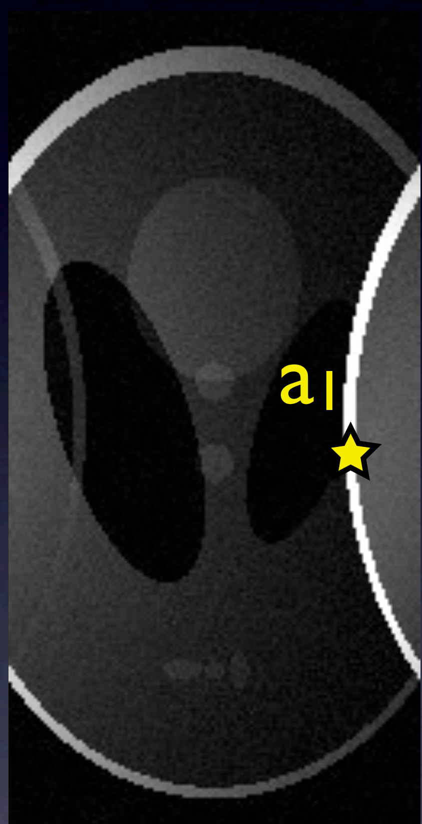
Undersampling



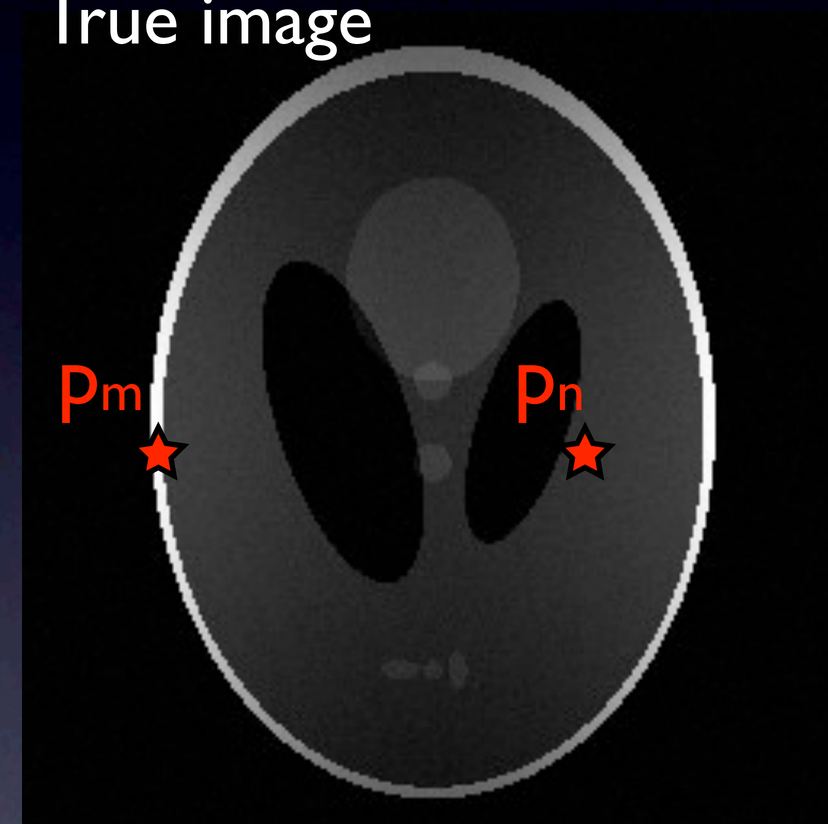
sub-Nyquist sampling

Magnetic Resonance Imaging

Undersampling



True image



Aliased signal

Coil sensitivity at ‘n’

$$a_l = S_{l,m}\, p_m + S_{l,n}\, p_n$$

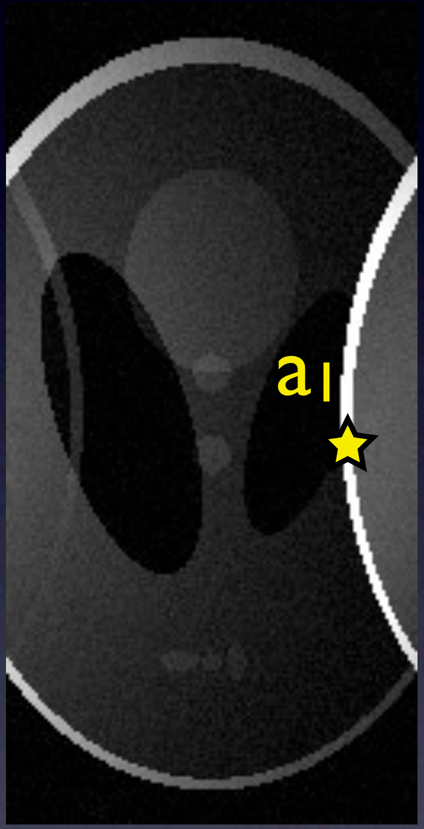
True MRI signal at ‘n’

Coil sensitivity at ‘m’

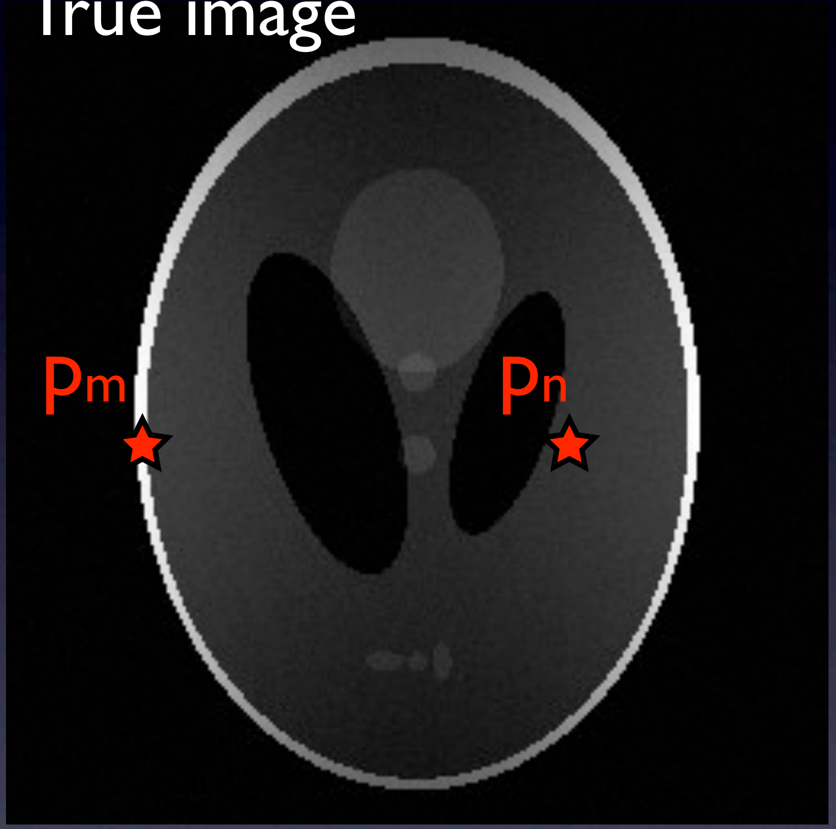
True MRI signal at ‘m’

Magnetic Resonance Imaging

Parallel Imaging



True image



$a_1 = S_{1,m}\, p_m + S_{1,n}\, p_n$

$a_2 = S_{2,m}\, p_m + S_{2,n}\, p_n$

In matrix form

$[A] = [S][P]$

...with solution:

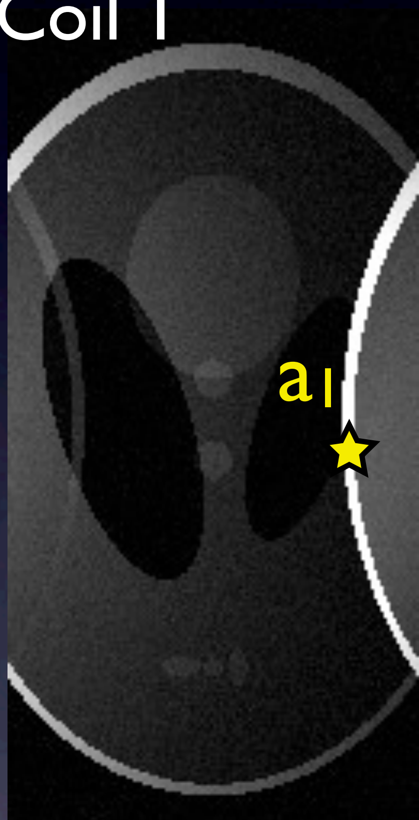
$P = (S^*S)^{-1}S^*A$

SENSE: Sensitivity Encoding
Pruessmann KP, et al. MRM 1999

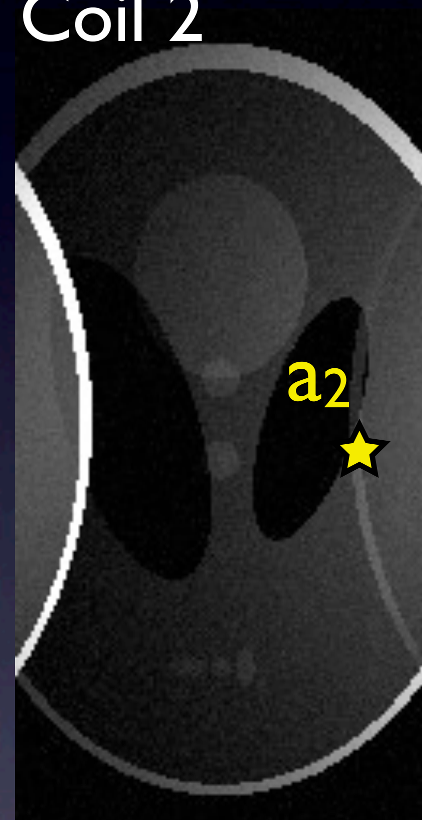
Magnetic Resonance Imaging

Parallel Imaging

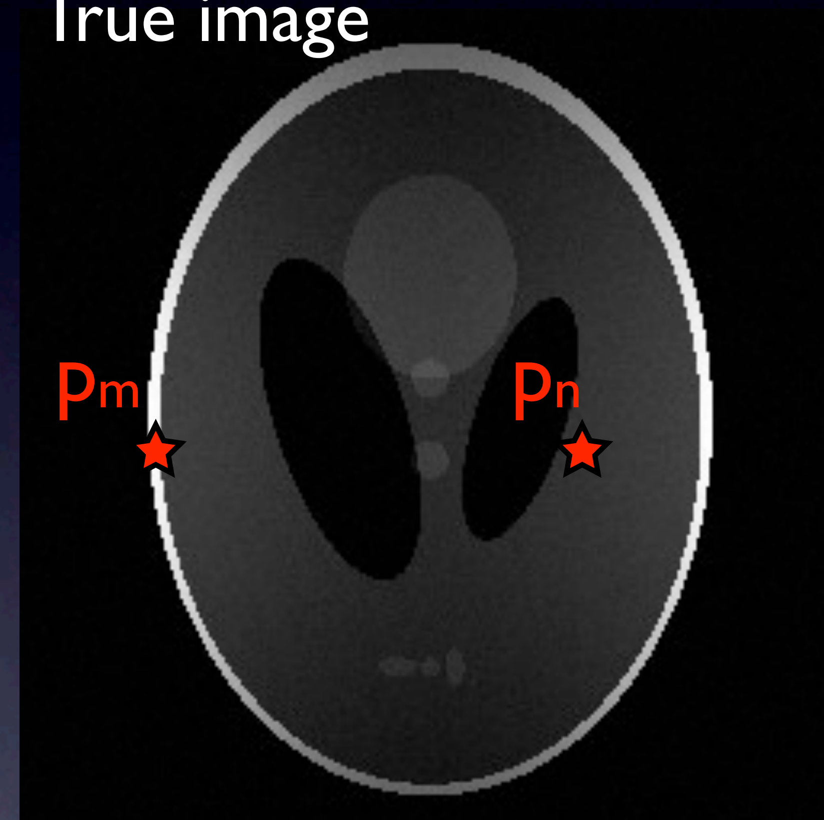
Coil 1



Coil 2



True image



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In matrix form

$$[A] = [S][P]$$

...with solution:

$$P = (S^*S)^{-1} S^* A$$

SENSE: Sensitivity Encoding
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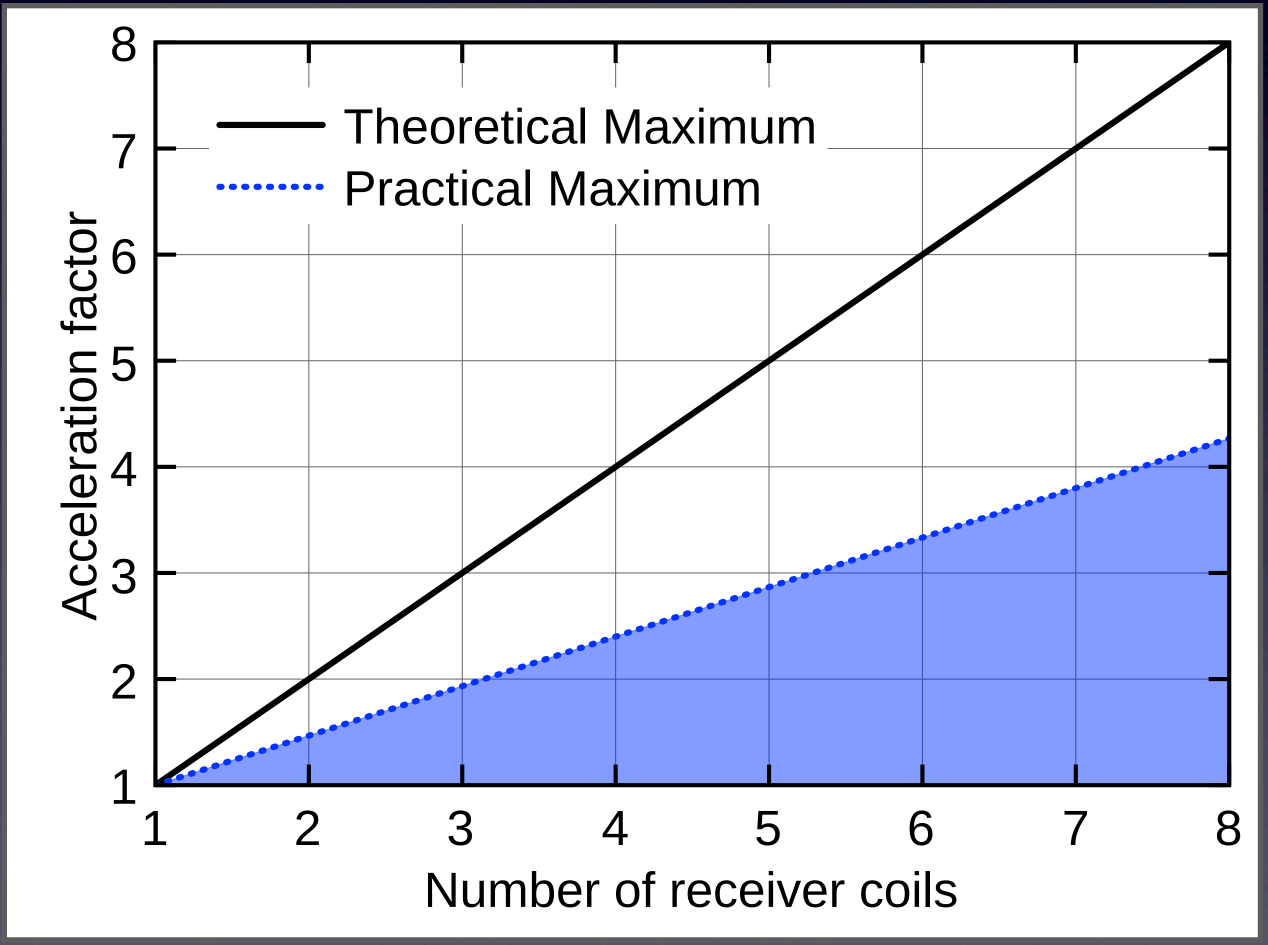
Magnetic Resonance Imaging

Parallel Imaging

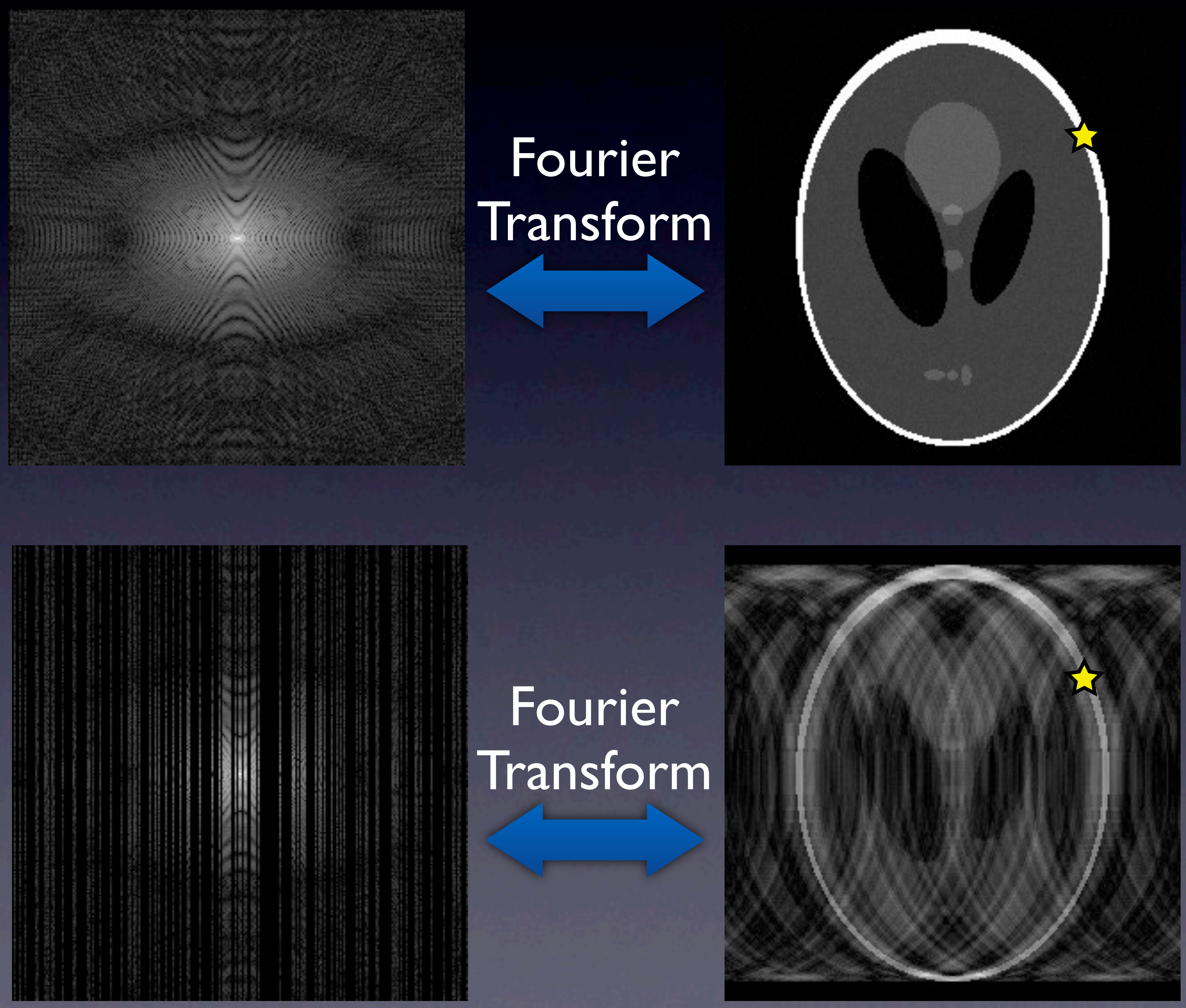
- Acceleration rate ≤ number of receiver coils
- Receiver coils must have a unique sensitivity along the accelerated dimensions



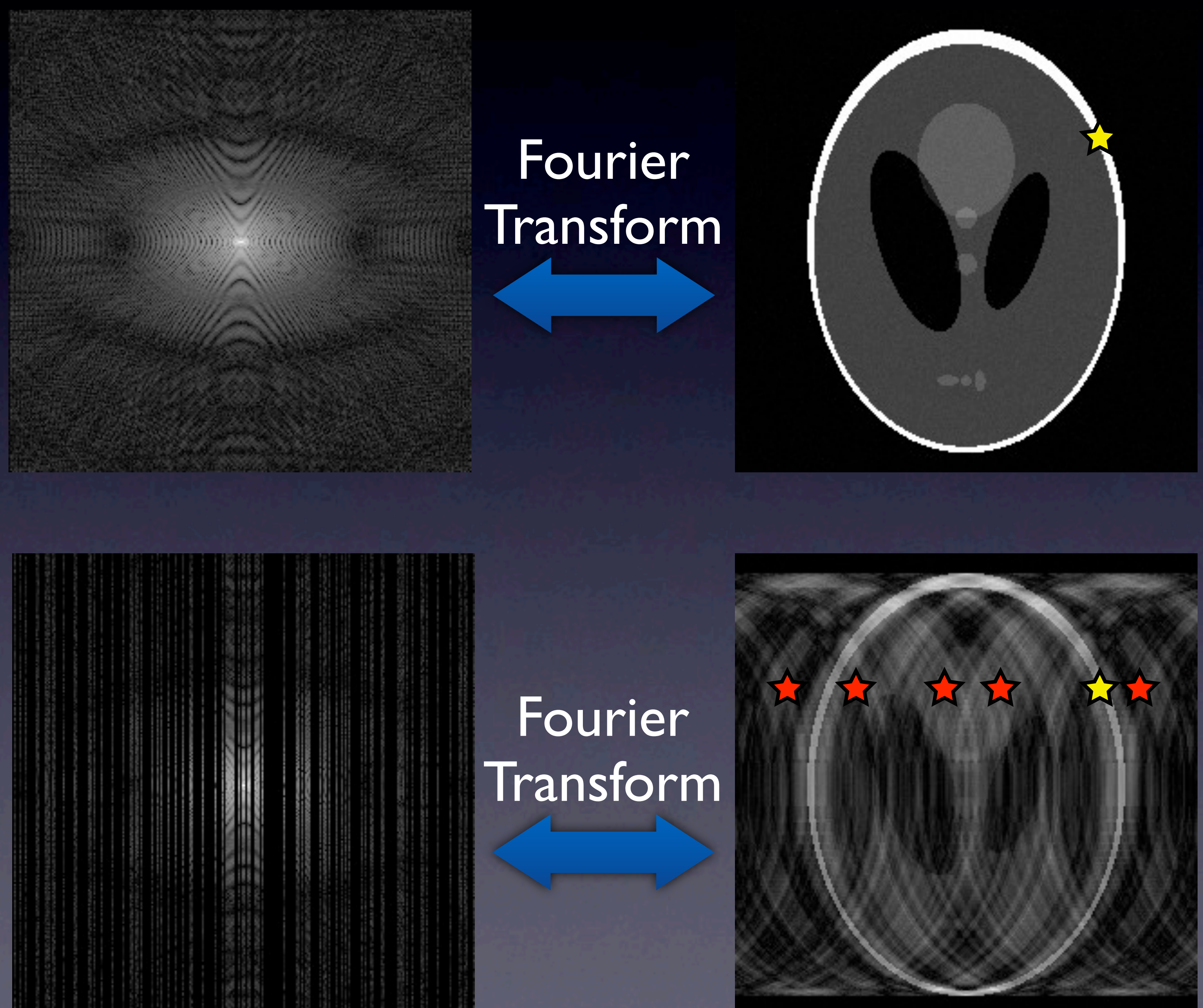
Recap

- MRI is insensitive and very slow
- Data is sampled in k-space
- Uniform undersampling produces *coherent* aliasing
  - Unwrapping is possible with multiple receiver coils (parallel imaging)

Random Undersampling

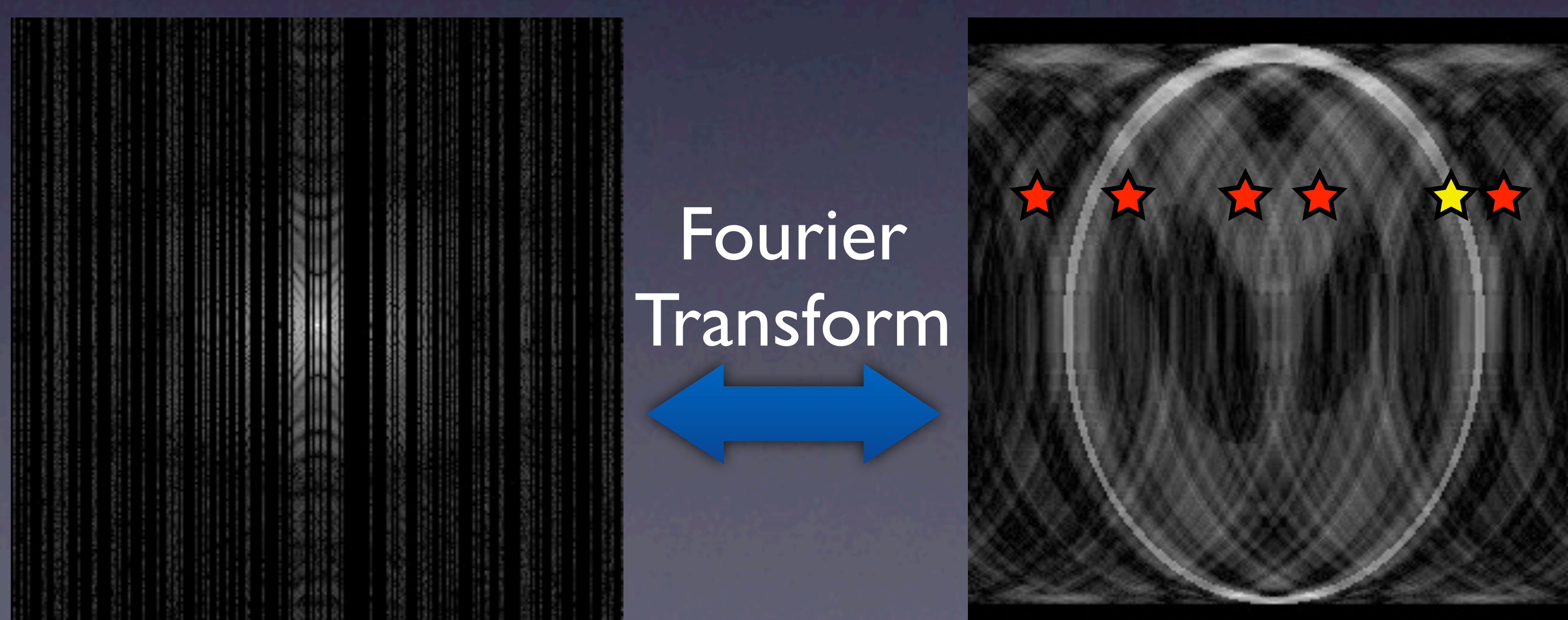


Random Undersampling

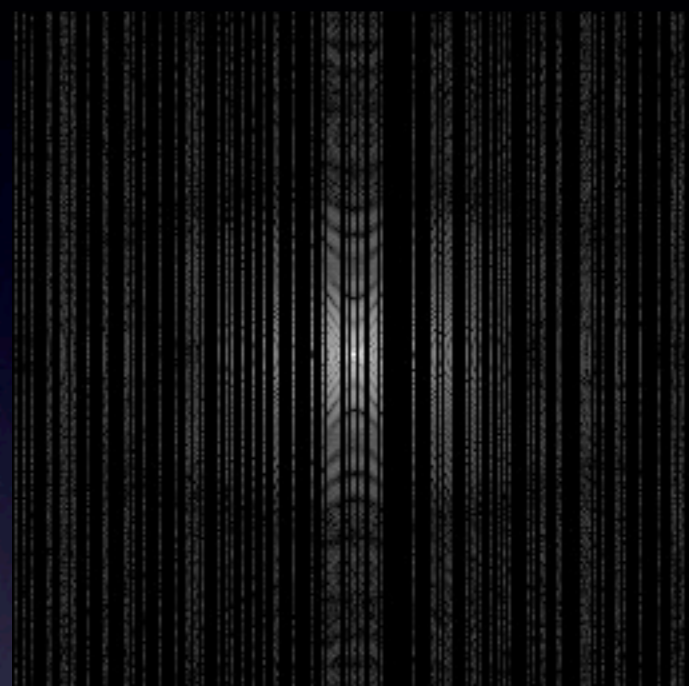


Random Undersampling

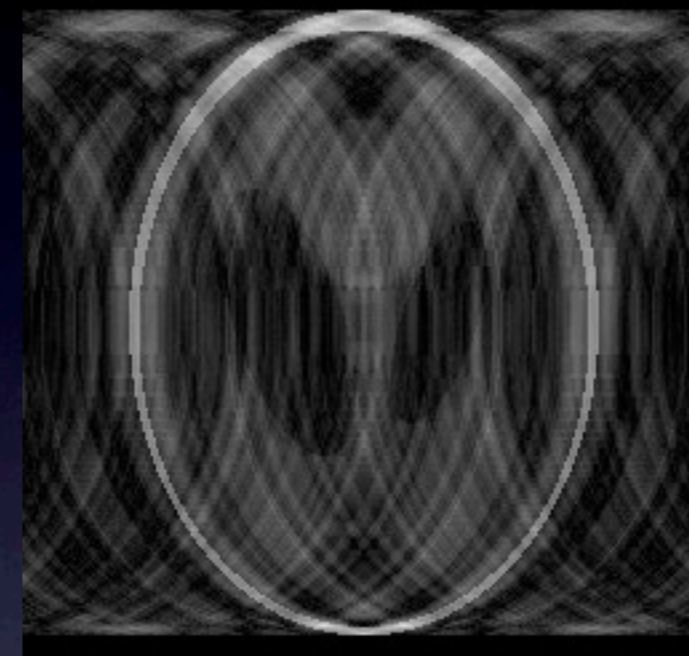
- T-e qu-st--n i-: h-w do w- s--pre-s -he inco--re-t -lias--g an- fi-l in th- -issing dat-?
- A--wer: c-nt-xt



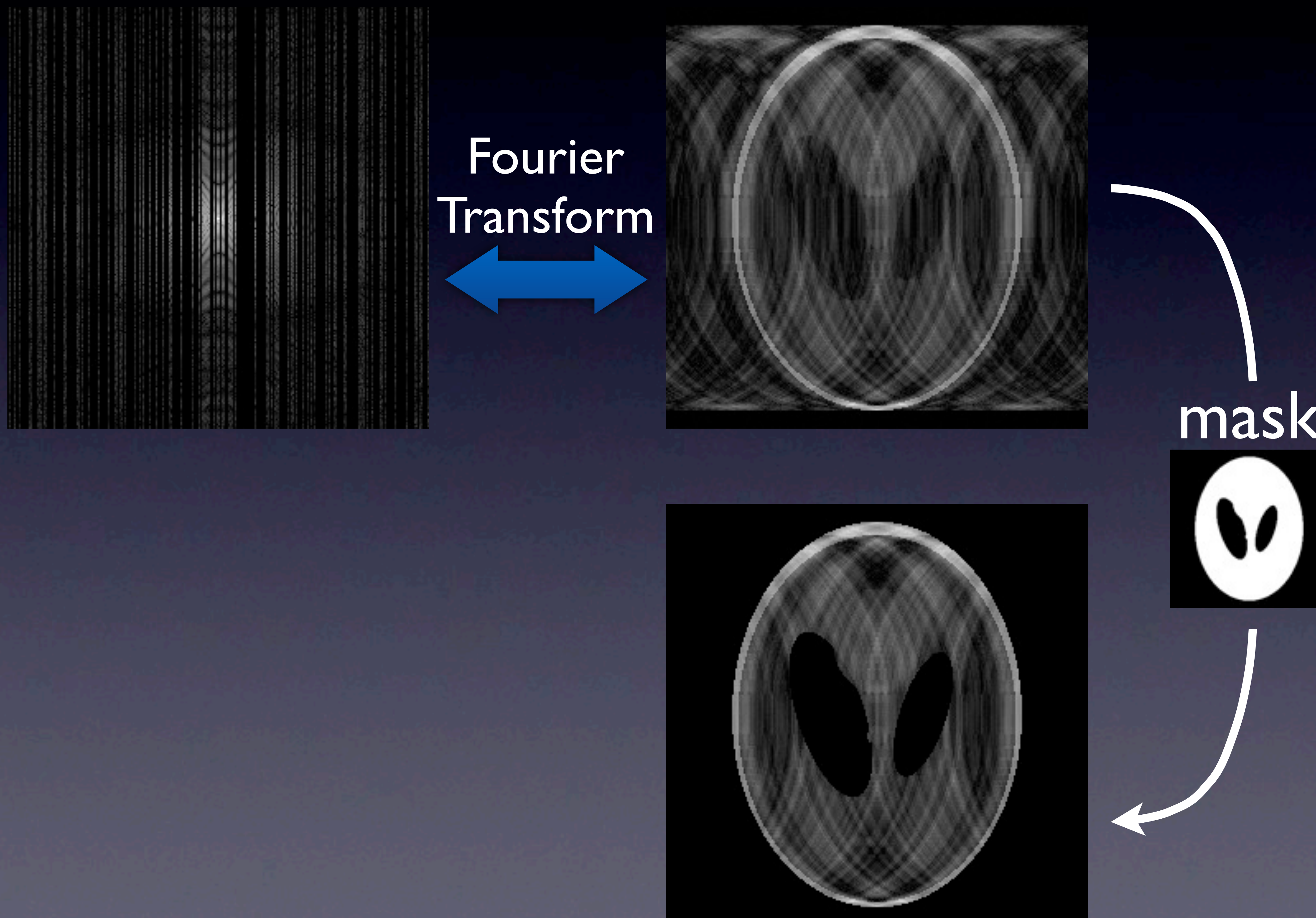
Constrained Reconstruction



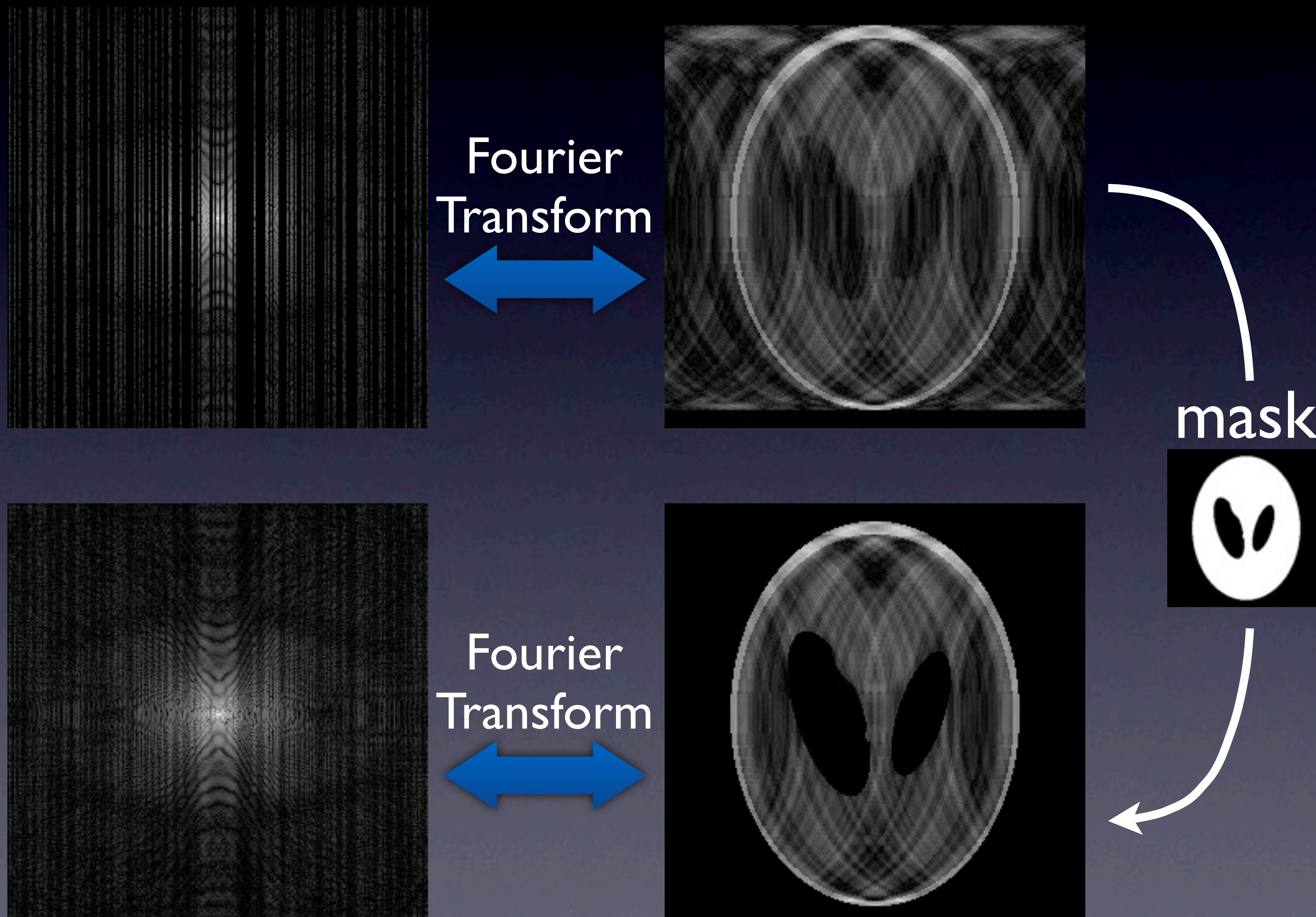
Fourier
Transform



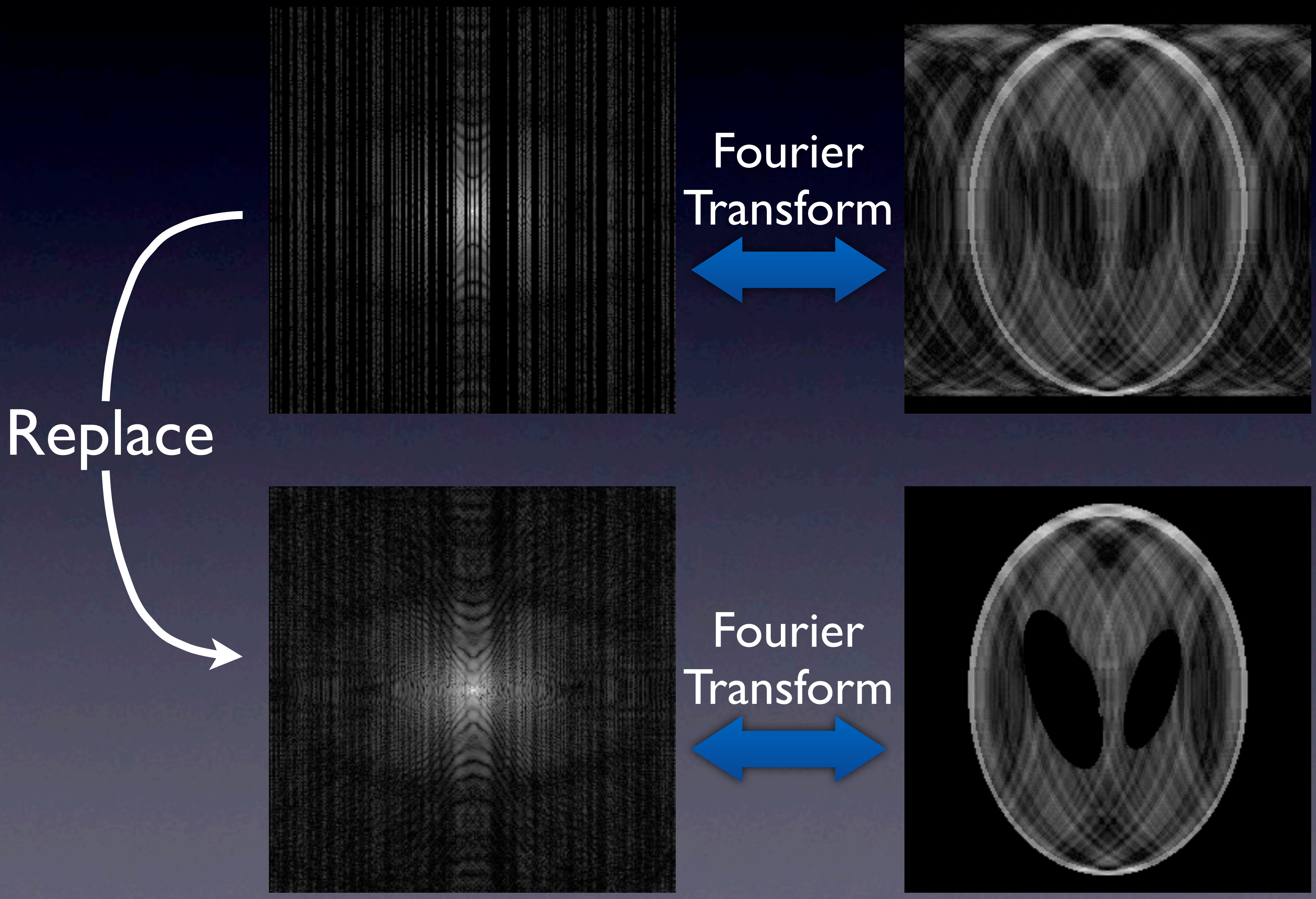
Constrained Reconstruction



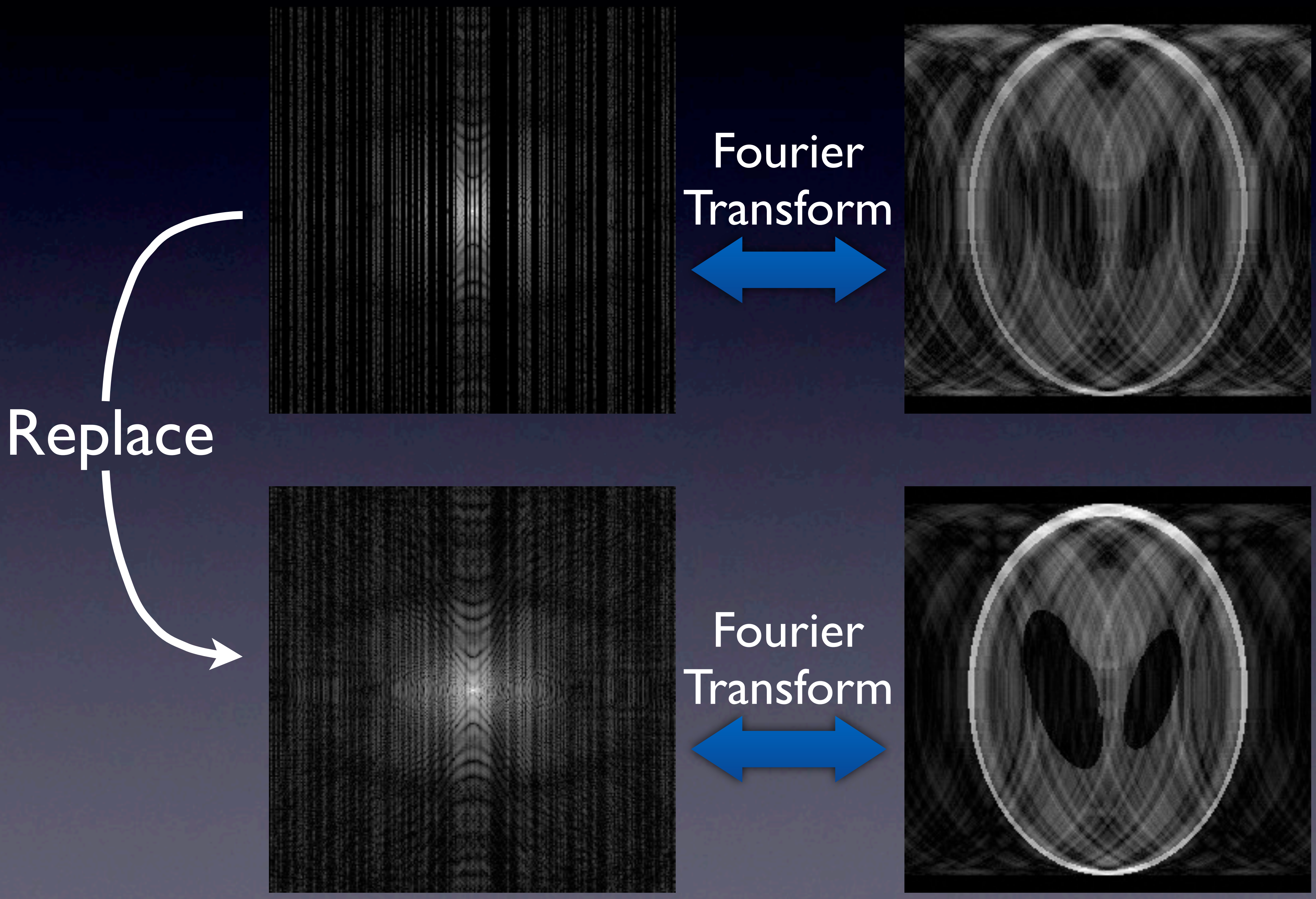
Constrained Reconstruction



Constrained Reconstruction



Constrained Reconstruction



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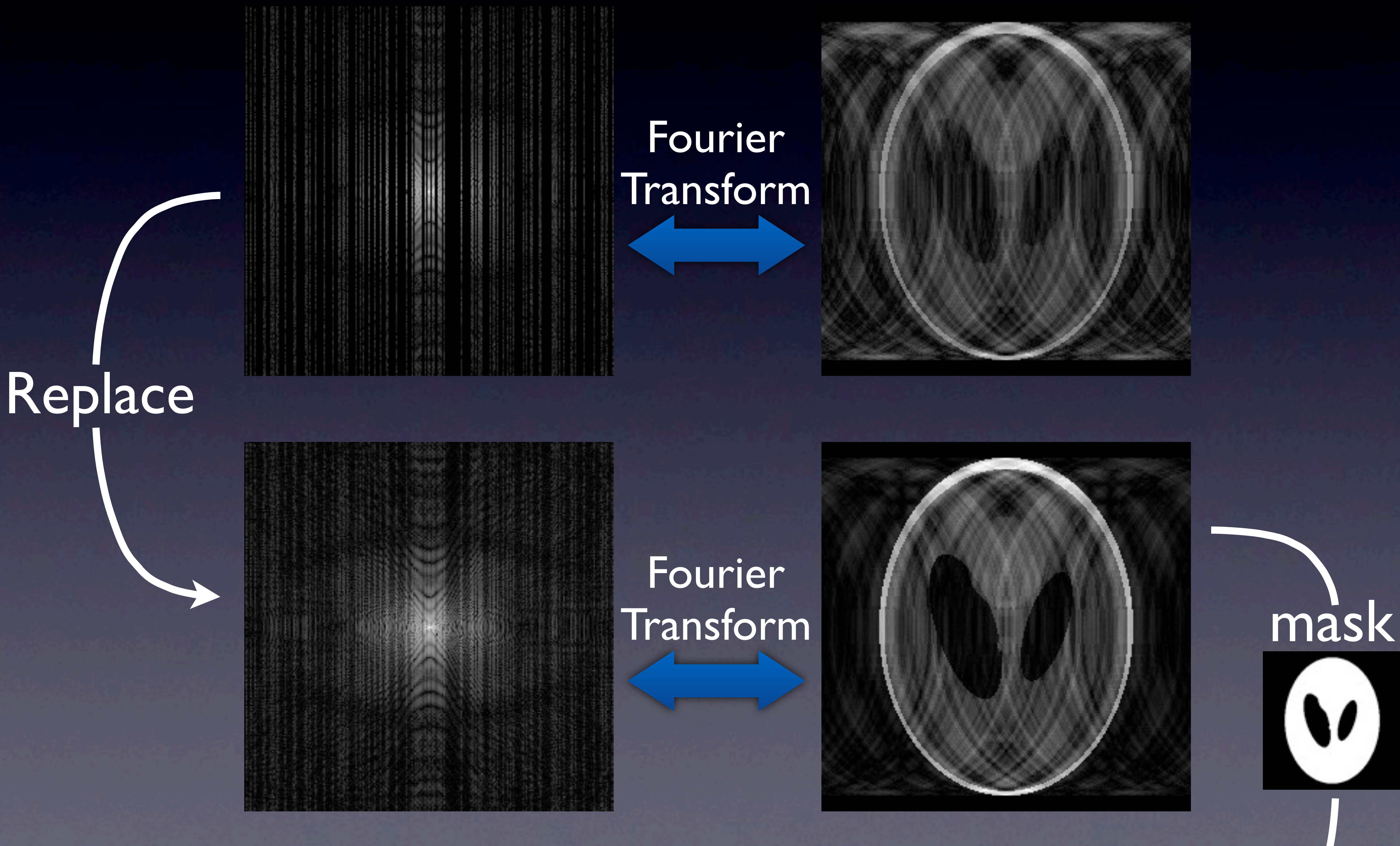

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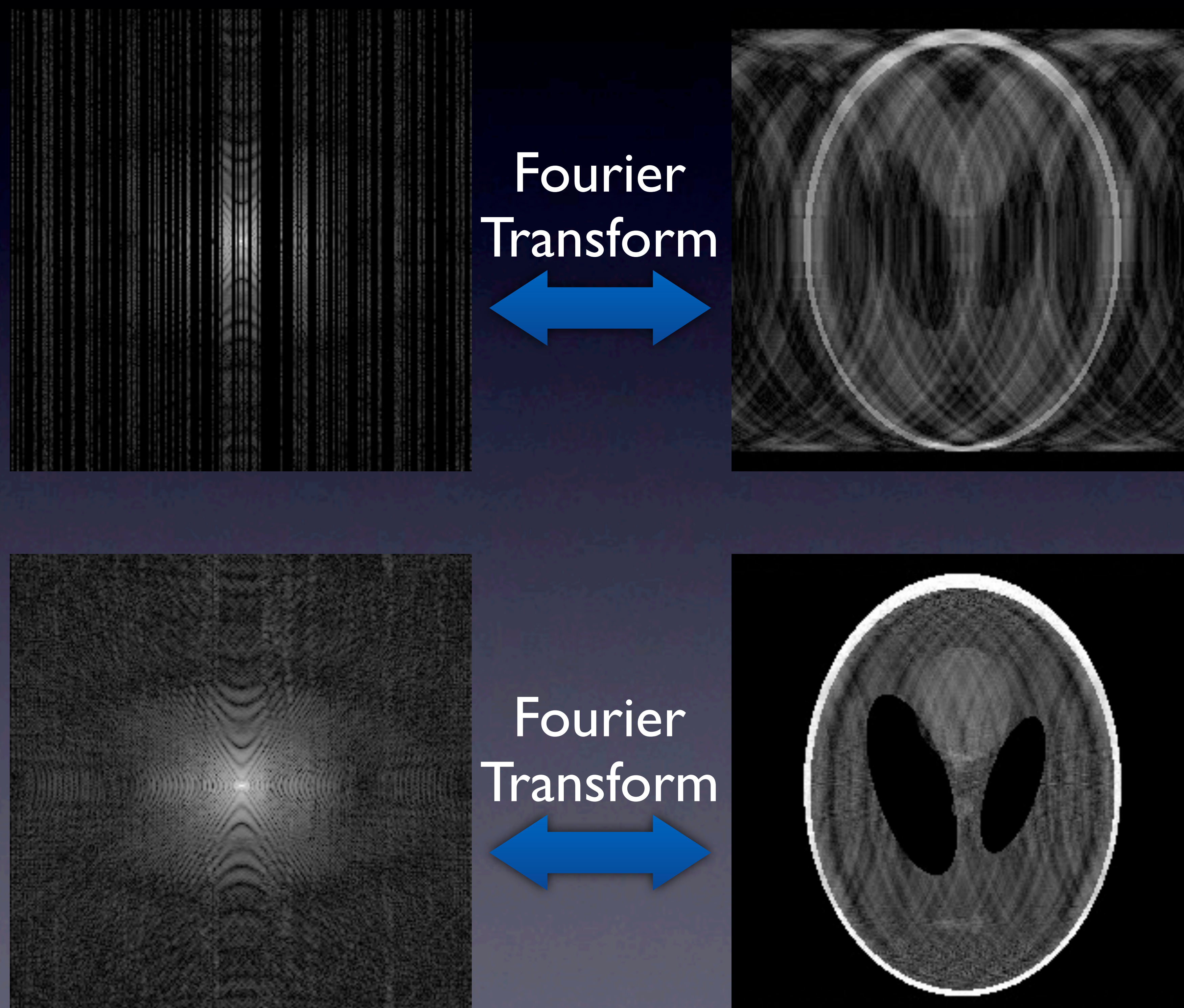

Iterate...

Constrained Reconstruction

- Initial observations:
  - Promote expected characteristic
  - Enforce data consistency
  - Iterate to improve the image

Mathematical Model

$$\left.\begin{array}{ll}\text{minimize} & ||\Psi m||_0\end{array}\right\}\ \text{Sparsity}$$

$$\left.\begin{array}{ll}\text{subject to} & ||\mathrm{F}m - y||_2^2 < \epsilon\end{array}\right\}\ \text{Data consistency}$$

Ψ	Sparsity Transform	m	Image	F	Fourier Transform	y	Acquired k-space data	ϵ	Noise threshold

- Two types of norms:

a) l_0: counts the number of coefficients

b) l_2: sum-of-squares

Mathematical Model

$$\text{minimize} \quad ||\Psi m||_0$$

$$\text{subject to} \quad ||\mathrm{F}m - y||_2^2 < \epsilon$$

Sparsity

Data consistency

Ψ Sparsity Transform

m Image

F Fourier Transform

y Acquired k-space data

ϵ Noise threshold

- Choose an image that:

  a) (mostly) conforms to the image model

  b) is consistent with the acquired data

Transform Sparsity

$$\Psi m$$

Ψ Sparsity Transform

m Image

- A mathematical operation that allows the image to be described with a small number of coefficients

Transform Sparsity

- Images are 'natural' (adjacent pixels are often correlated)
  - Wavelet Transform Ψm

Image domain

Wavelet domain

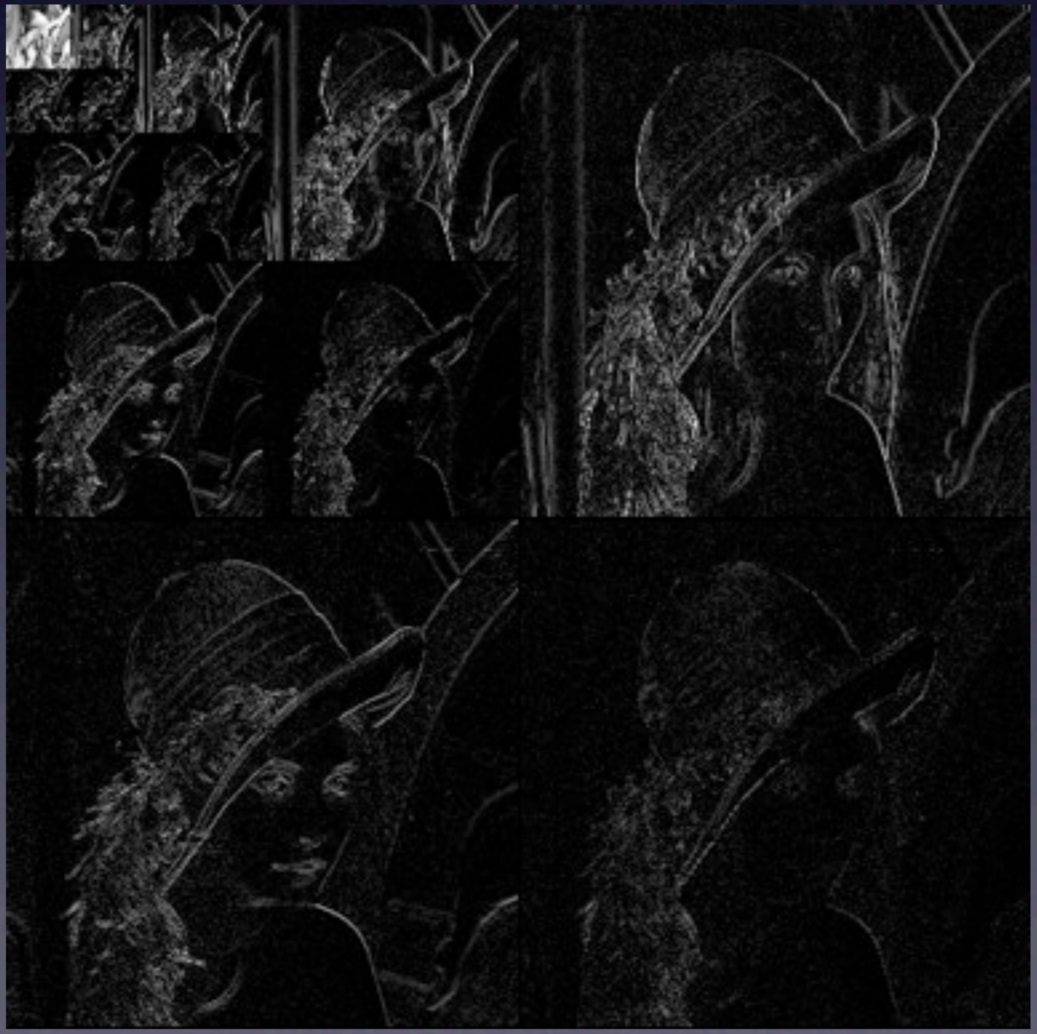

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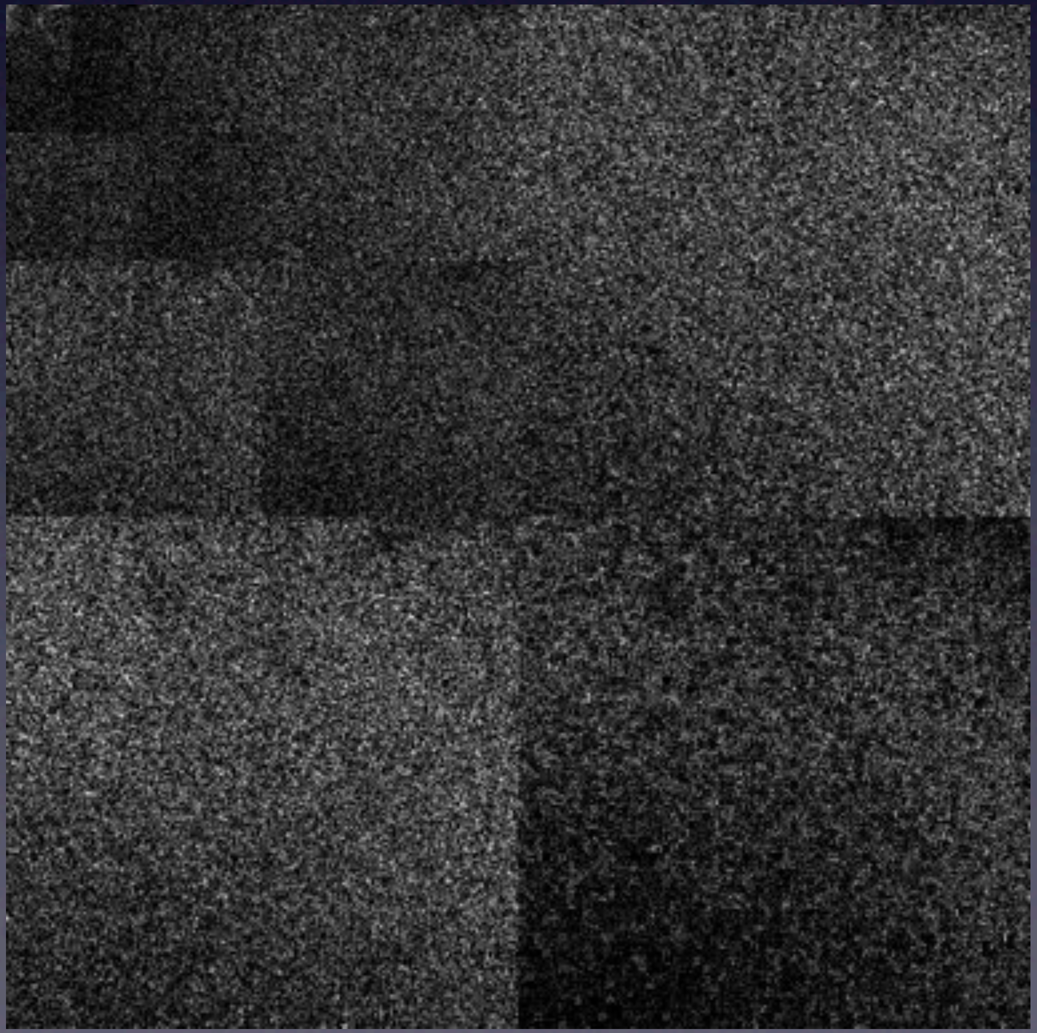
Image domain

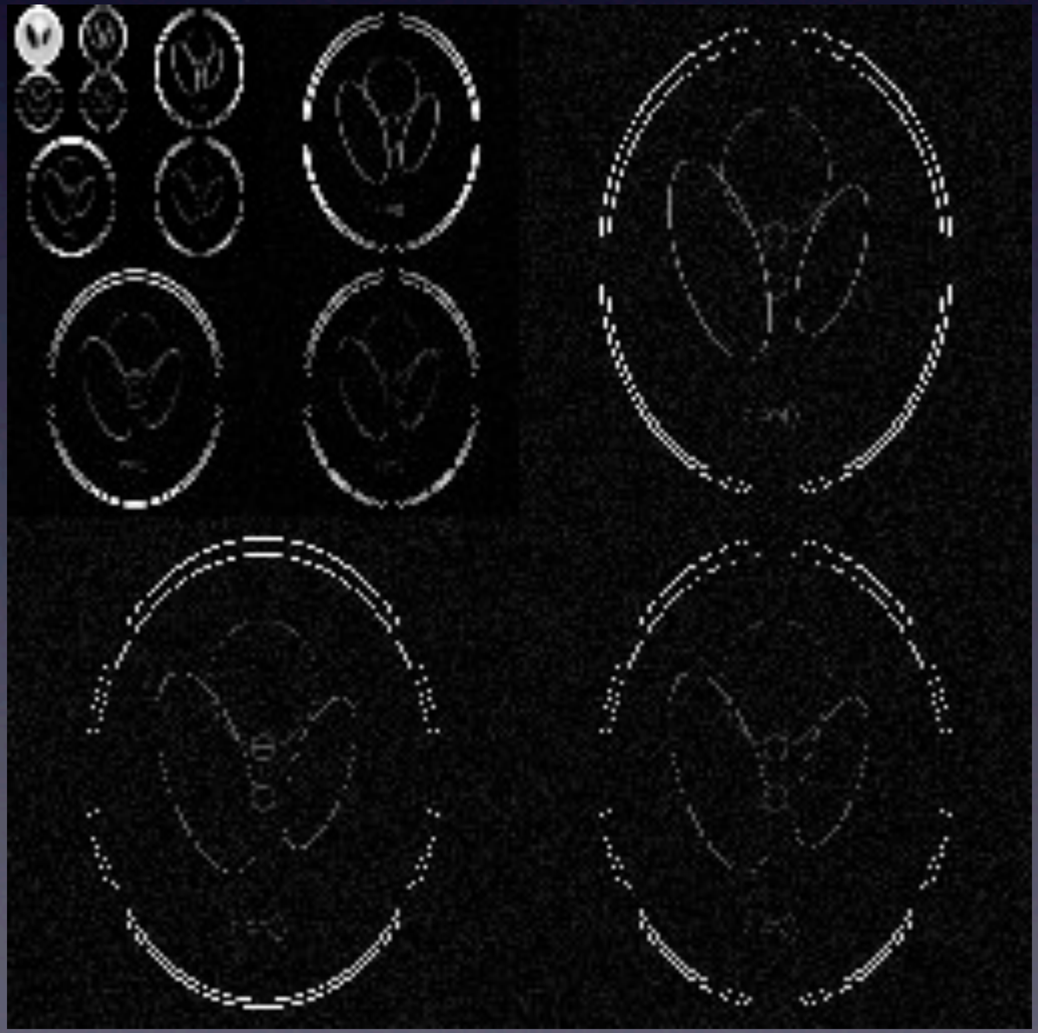
Wavelet domain



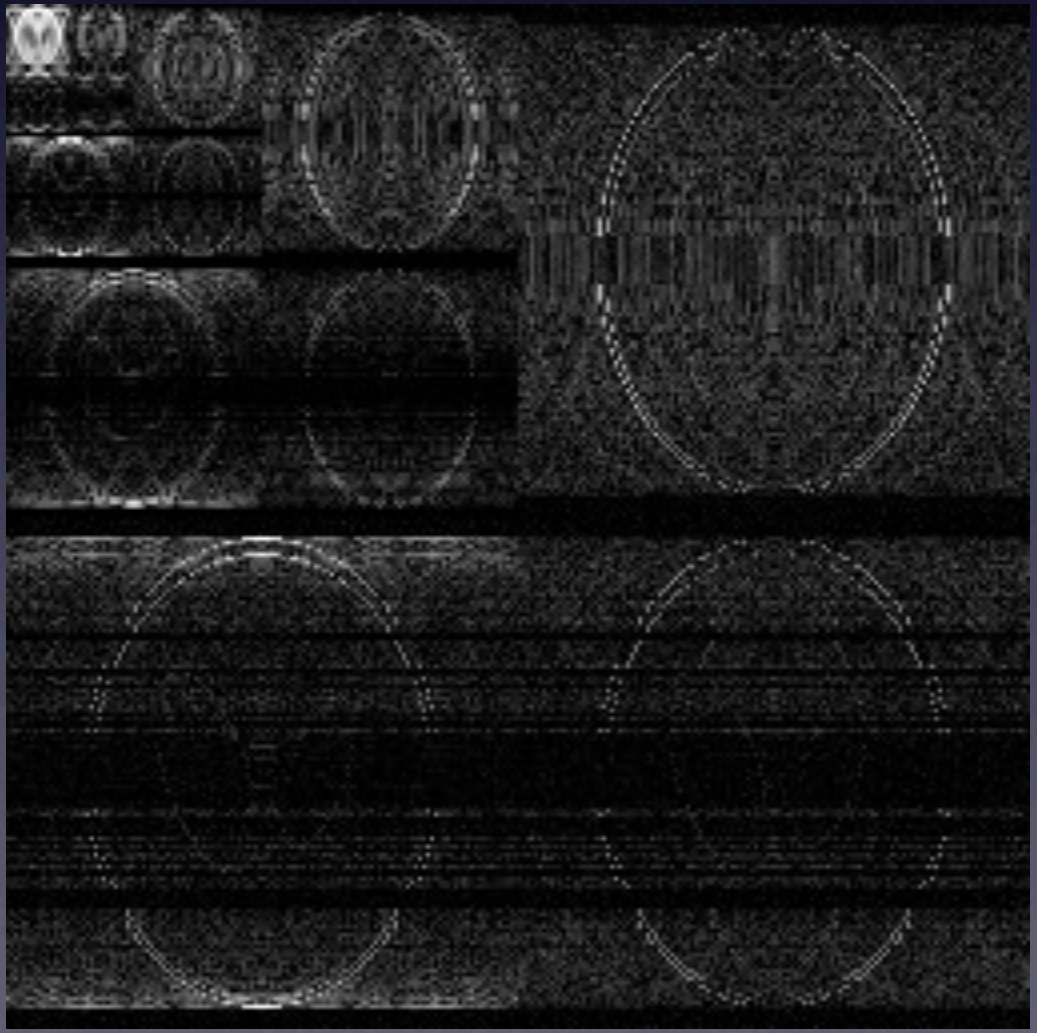
Transform Sparsity

- Images are 'natural' (adjacent pixels are often correlated)
  - Wavelet Transform Ψm

Fully sampled



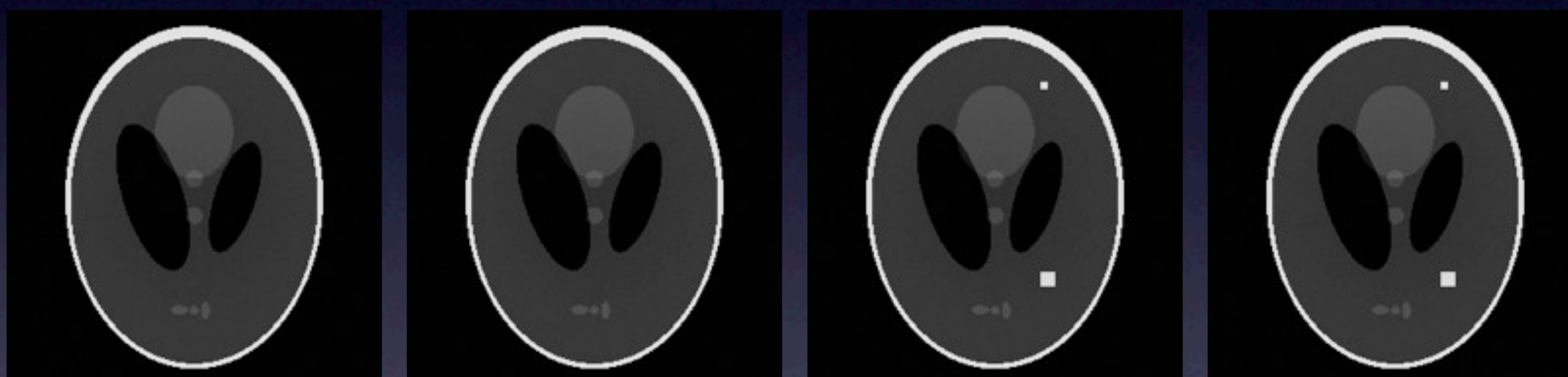
Undersampled



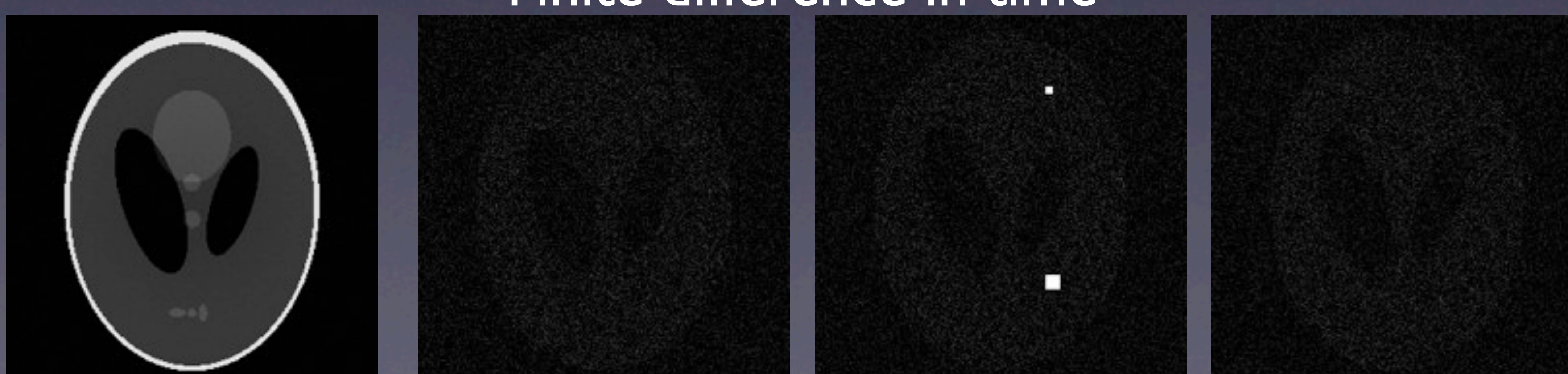
Transform Sparsity

- Dynamic images do not change very much

time frame: 1 2 3 4



Finite difference in time



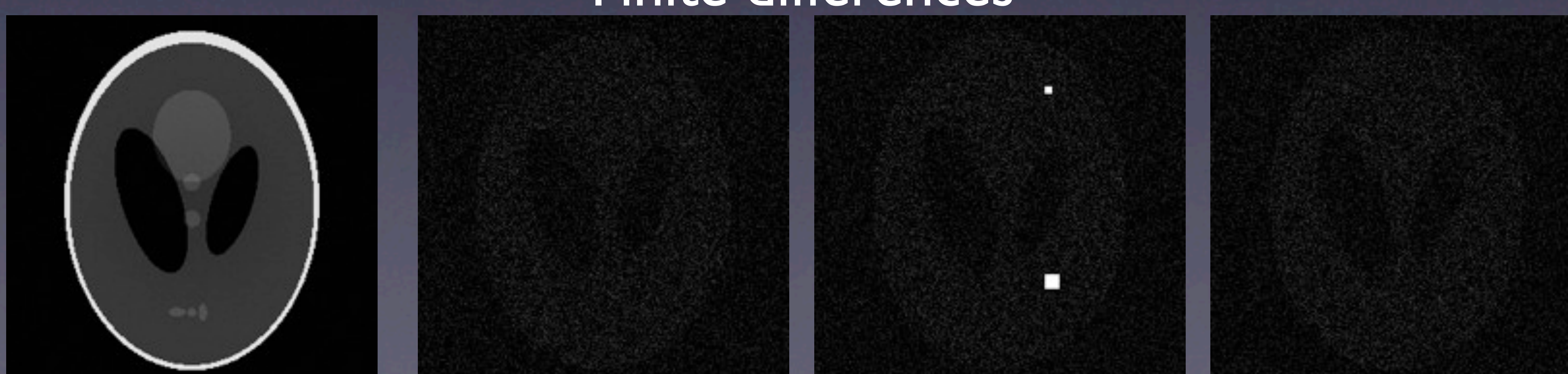
Transform Sparsity

- Dynamic images do not change very much

$$(\mathrm{T} - \mathrm{I})m$$

T Time shift I Identity

Finite differences



Mathematical Model

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Ψ Sparsity Transform

m Image

F Fourier Transform

y Acquired k-space data

ϵ Noise threshold

- Choose an image that:

  a) (mostly) conforms to the image model

  b) is consistent with the acquired data

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Ψ Sparsity Transform

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F Fourier Transform

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ϵ Noise threshold

- This problem is very difficult to solve
  - It is non-convex (may have local minima)

Mathematical Model

$$\min_{m} \left\{ ||\mathrm{F}m - y||_2^2 + \lambda \, ||\Psi m||_1 \right\}$$

Ψ Sparsity Transform

m Image

F Fourier Transform

y Acquired k-space data

ϵ Noise threshold

λ Regularization factor

- Convert the l_0 norm to an l_1 norm
  - Sum of absolute values
  - Still promotes sparsity

Mathematical Model

$$\min_m \left\{ ||\mathrm{F}m - y||_2^2 + \lambda \, ||\Psi m||_1 \right\}$$

Ψ Sparsity Transform

m Image

F Fourier Transform

y Acquired k-space data

ϵ Noise threshold

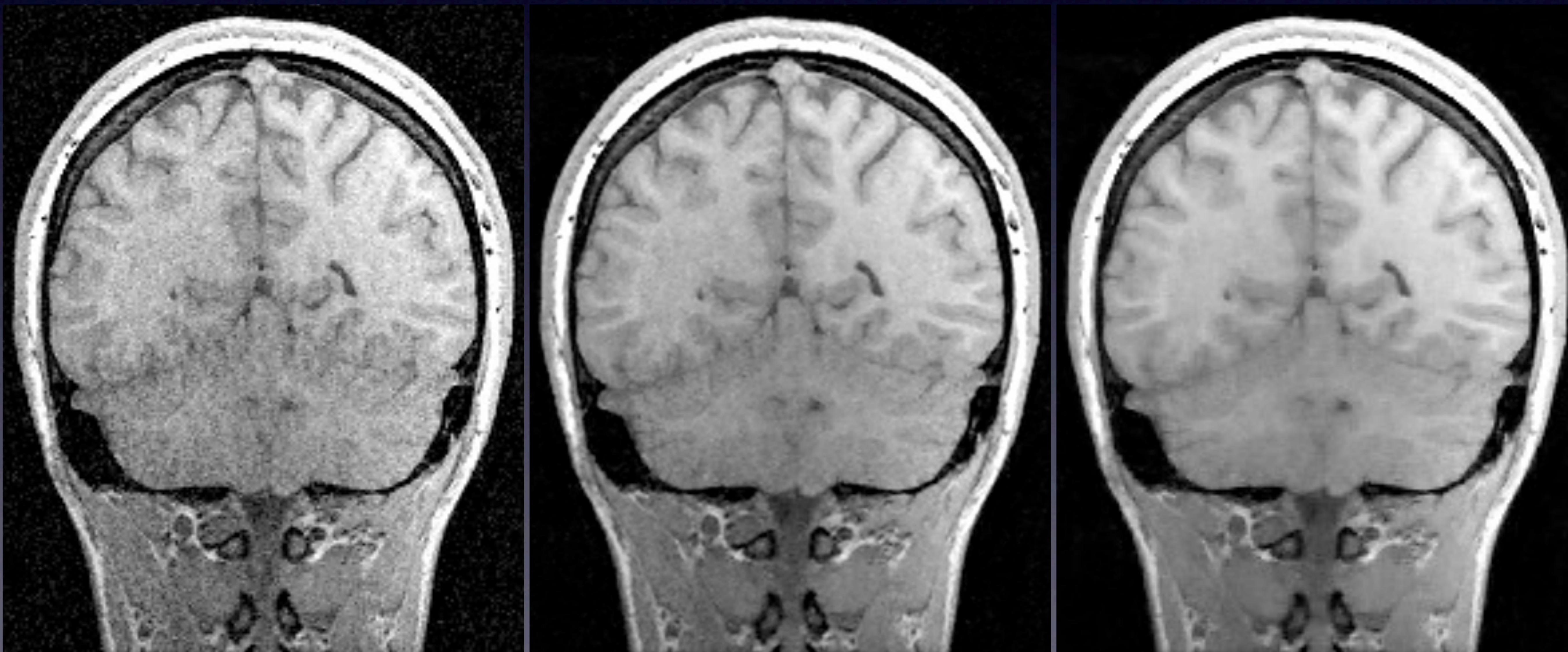
λ Regularization factor

- This problem is convex (no local minima)
  - Conjugate gradient
  - Thresholding techniques
- Solution is iterative (very slow!)
- Must choose λ

Regularization factor

$$\min_{m} \left\{ ||\mathrm{F}m - y||_2^2 + \lambda \, ||\Psi m||_1 \right\}$$

λ too small? λ just right? λ too big?



Reconstruction Movie

k-space

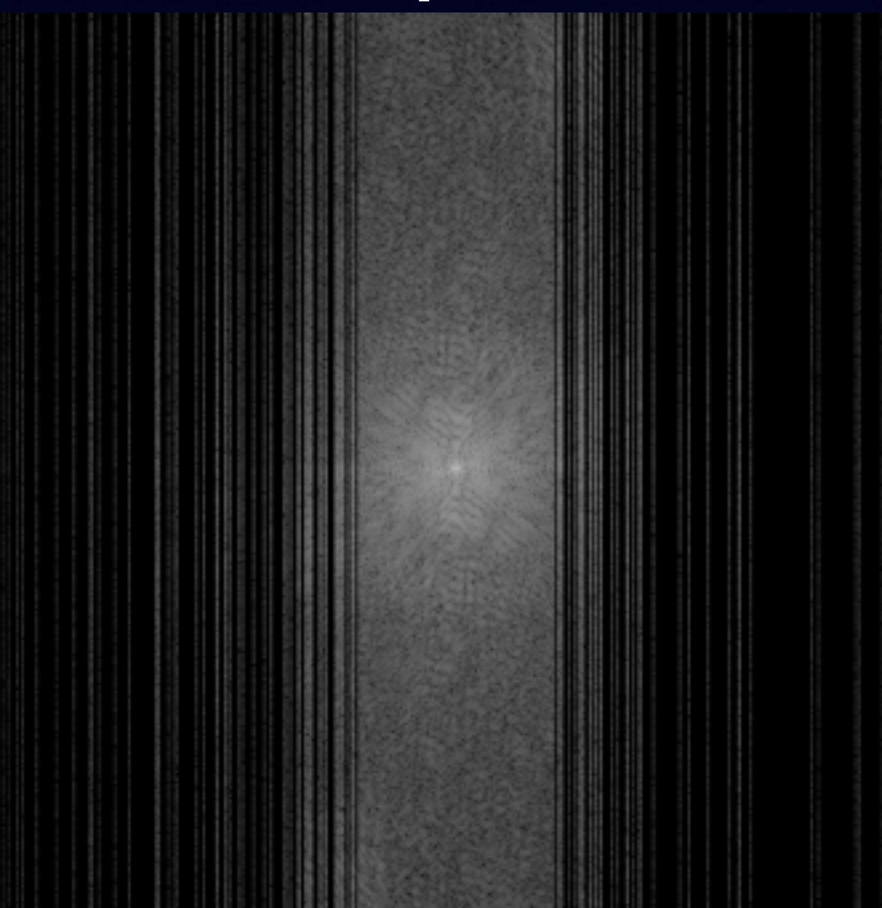
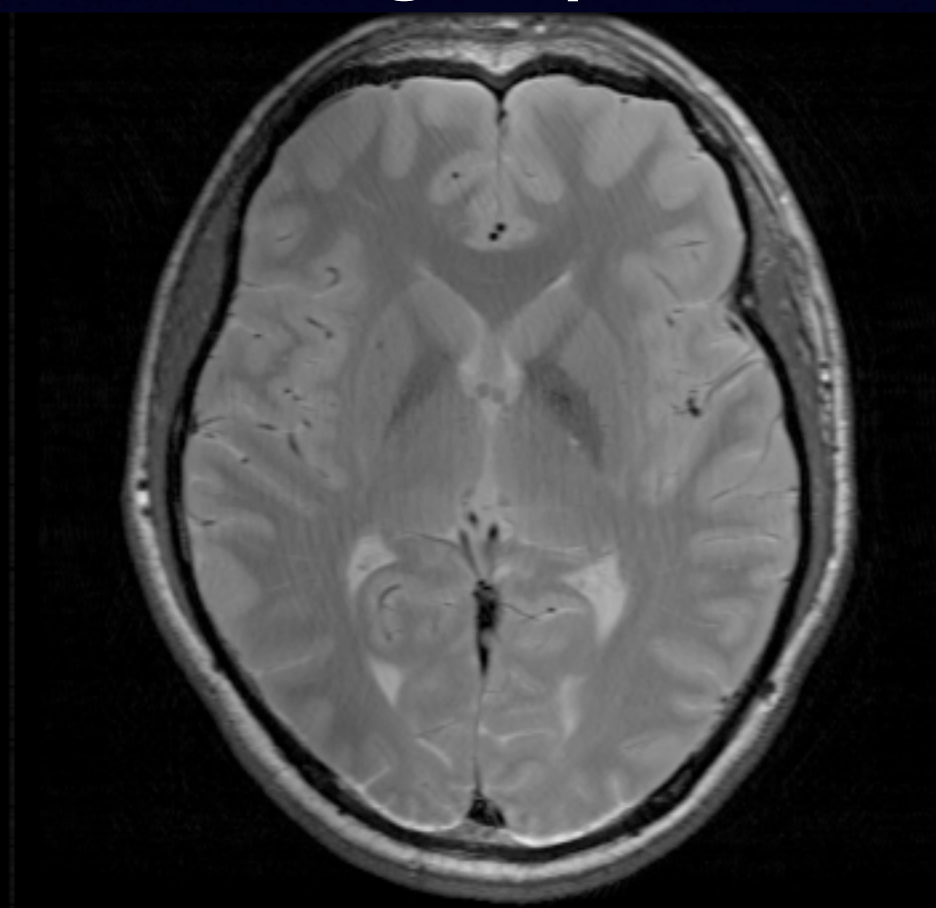
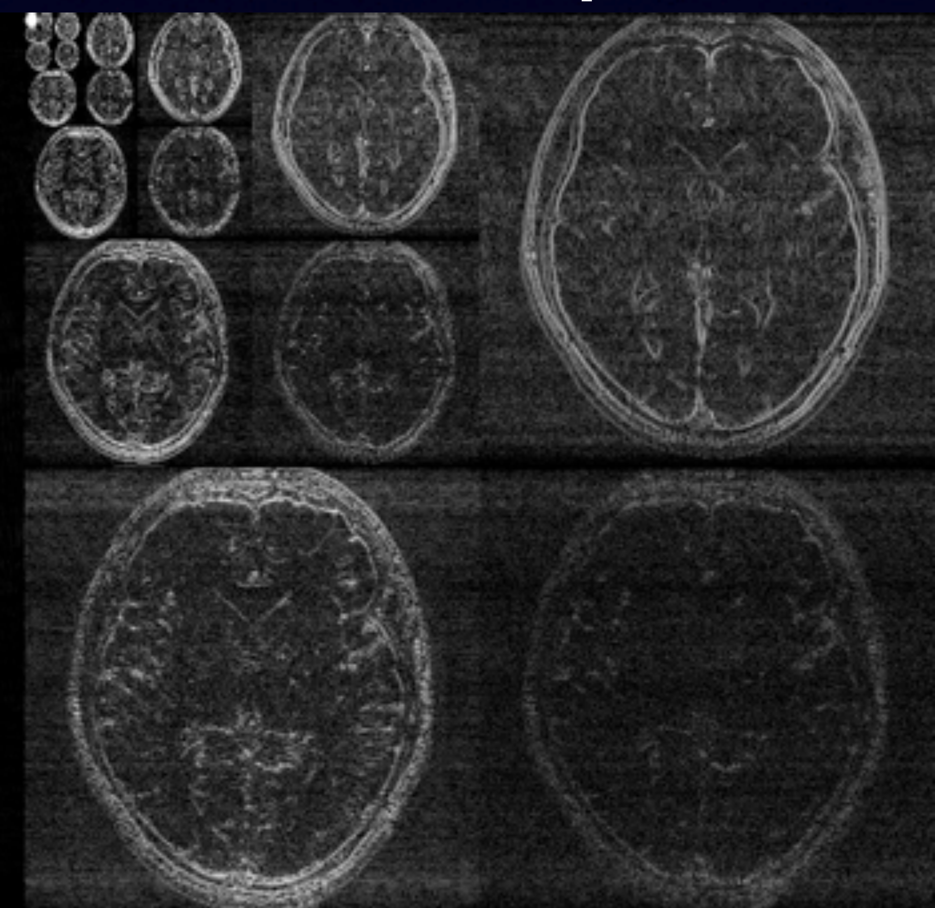


image-space



wavelet-space

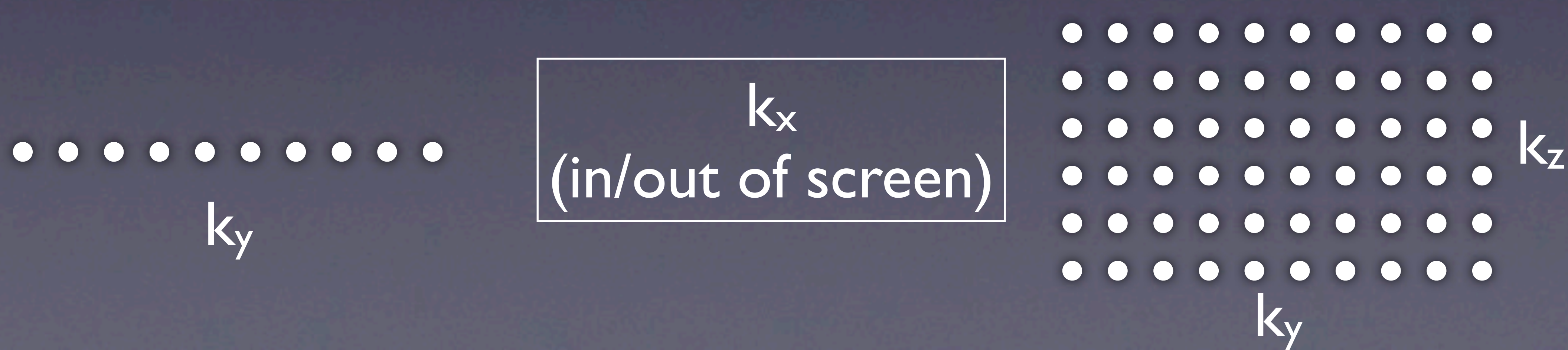


Sampling Patterns

- Avoid coherent aliasing in sparsity domain
  - Pseudo-random sampling
- Undersample in domains/dimensions that take time to acquire
  - Often phase-encode dimensions in k-space
- Undersample in as many dimensions as possible

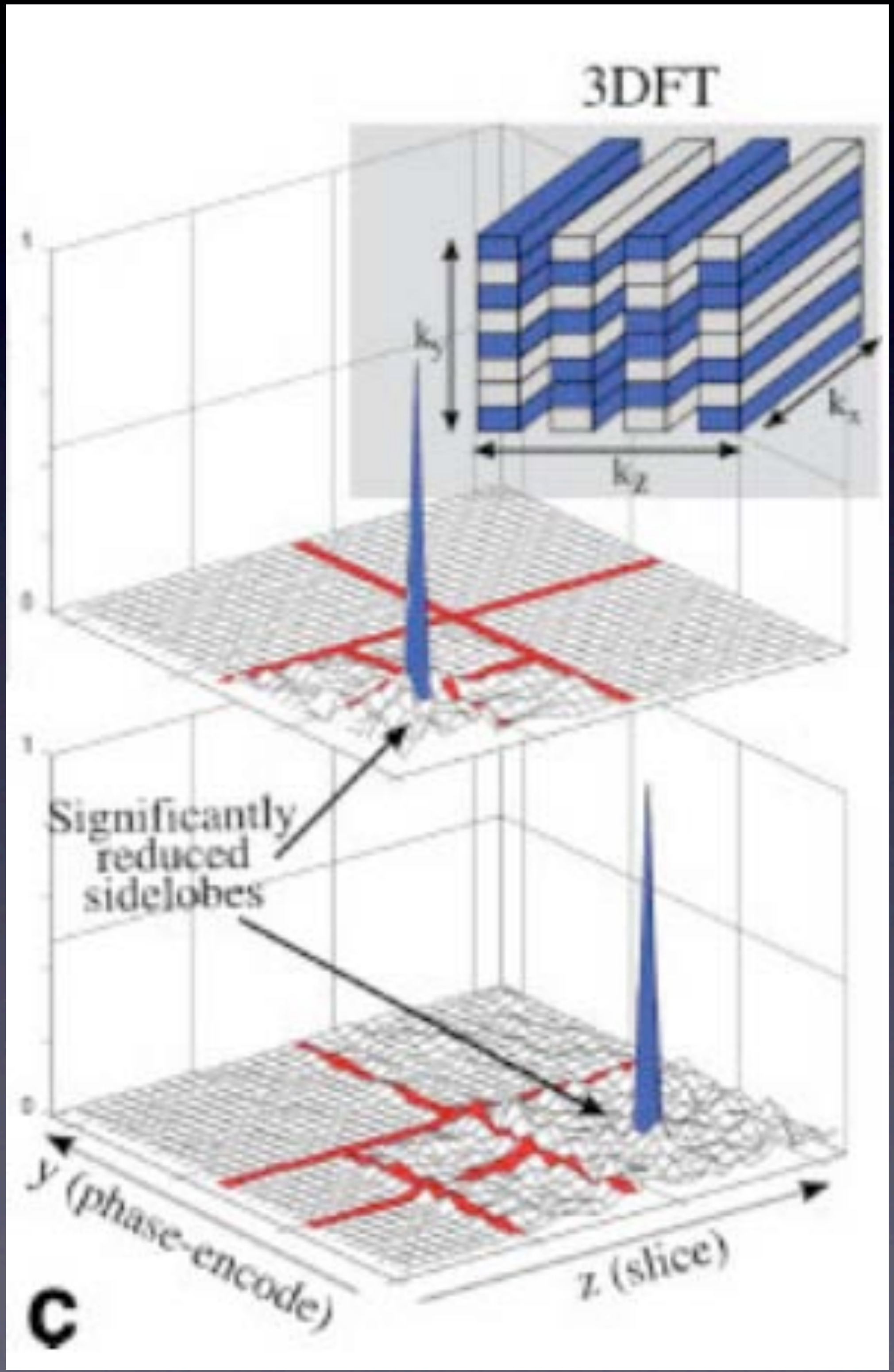
2D → 1D undersampling

3D → 2D undersampling



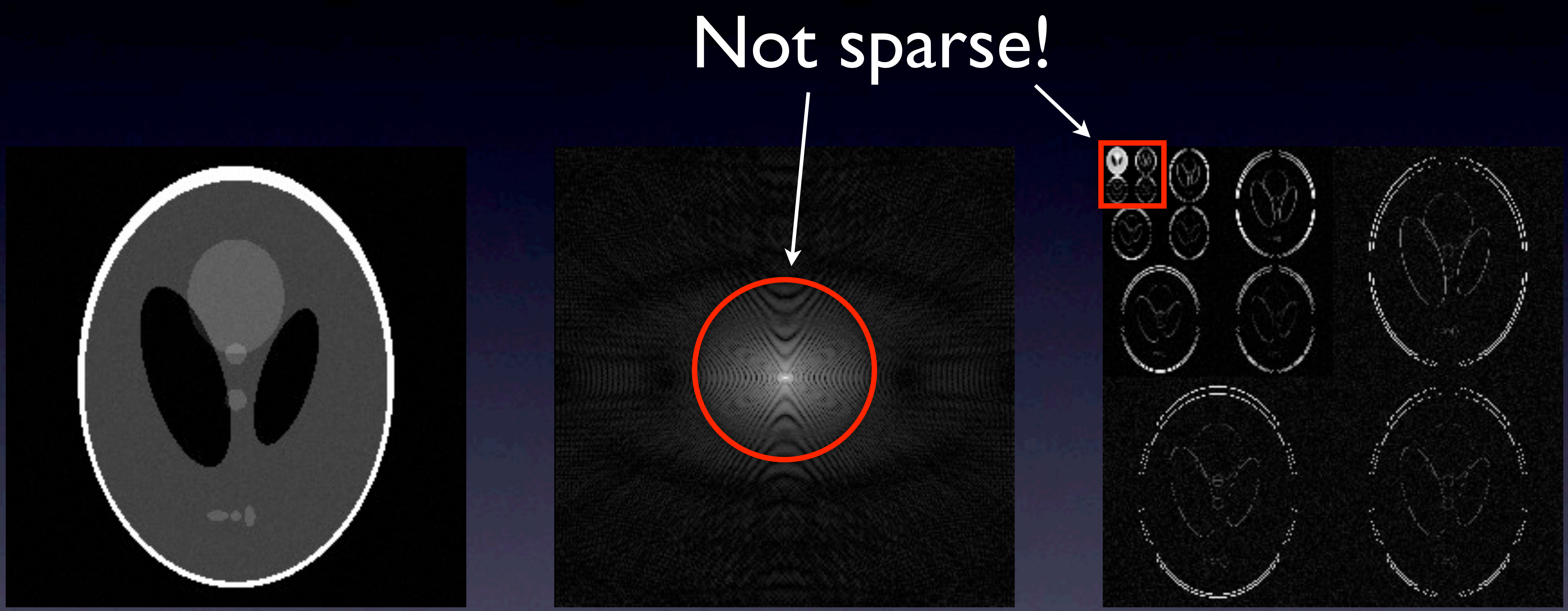
Sampling Patterns

- Avoid coherent aliasing in sparsity domain
  - Pseudo-random sampling
  - Can test several different patterns



M. Lustig *et al.* MRM 2007

Sampling Patterns



- The center of k-space (describes the low resolution image) has a lot of energy
- It should be sampled more heavily

Sampling Patterns

- Variable density sampling compensates for the lack of sparsity at low resolutions

Central lines: 0 8 16 32 48 85

MSE: 24.2 14.2 8.98 4.22 4.06 8.44

Compressed Sensing and Parallel Imaging

l_1-SPIRiT

(Iterative Self-consistent Parallel Imaging Reconstruction)

Lustig M. MRM 2010

$$\min_m \left\{ ||\mathrm{F}m - y||_2^2 + \lambda_0 ||(\mathrm{G} - \mathrm{I})m||_2 + \lambda_1 \, ||\Psi m||_1 \right\}$$

CS-SENSE

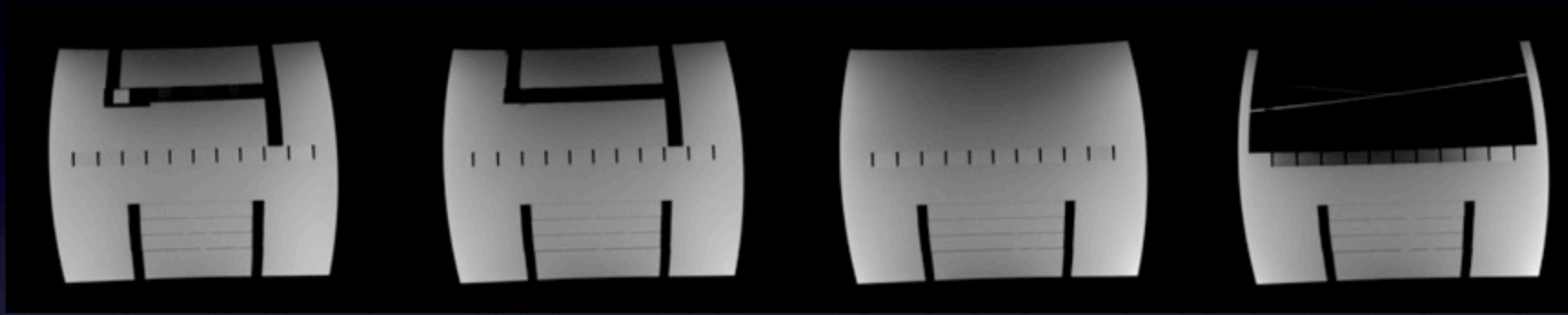
$$\min_m \left\{ ||\mathrm{FS}m - y||_2^2 + \lambda \, ||\Psi m||_1 \right\}$$

Recent work from MREL

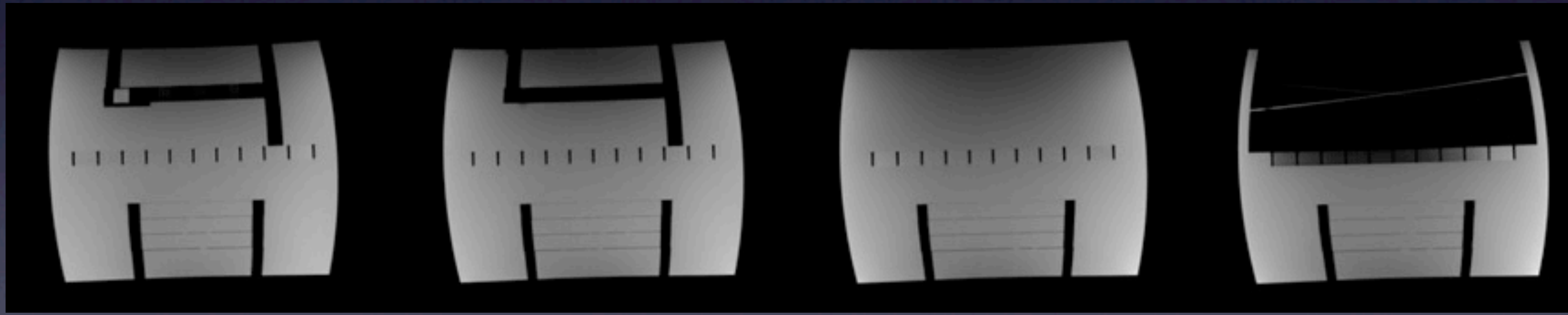
- We have been working to implement compressed sensing and parallel imaging on the scanner
  - GE 3T scanners at HSC
- Fat/water separation
- Visualization of the upper airway
- Neuroimaging

Phantom images

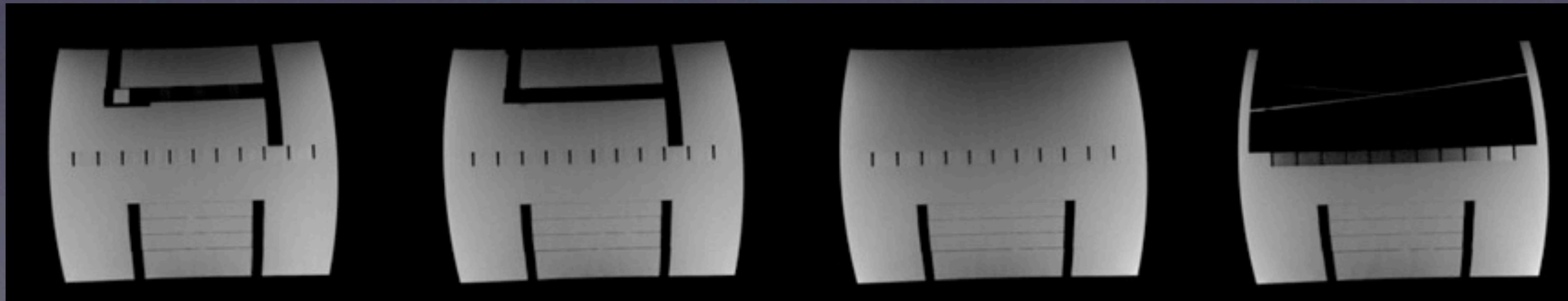
1.0x



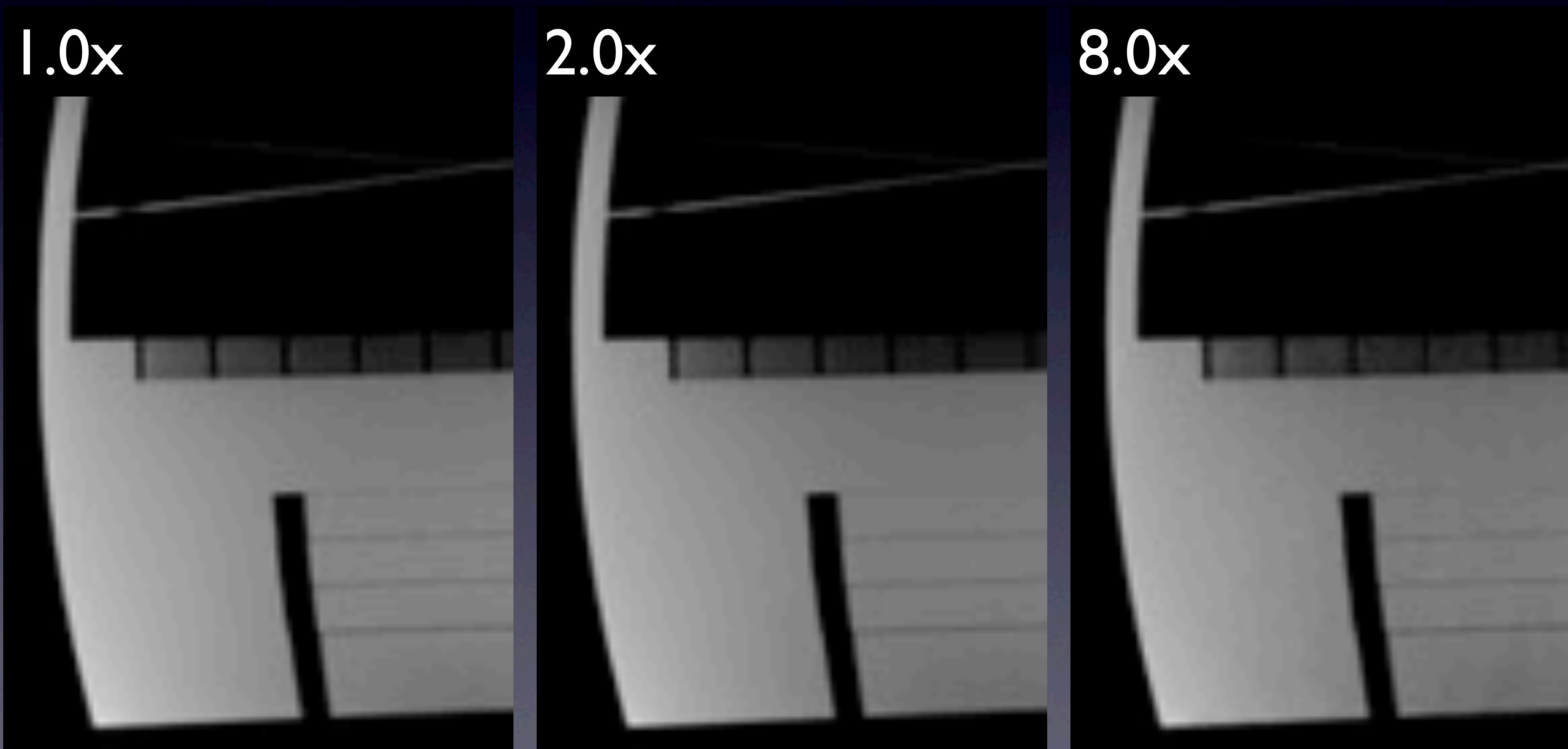
2.0x



8.0x

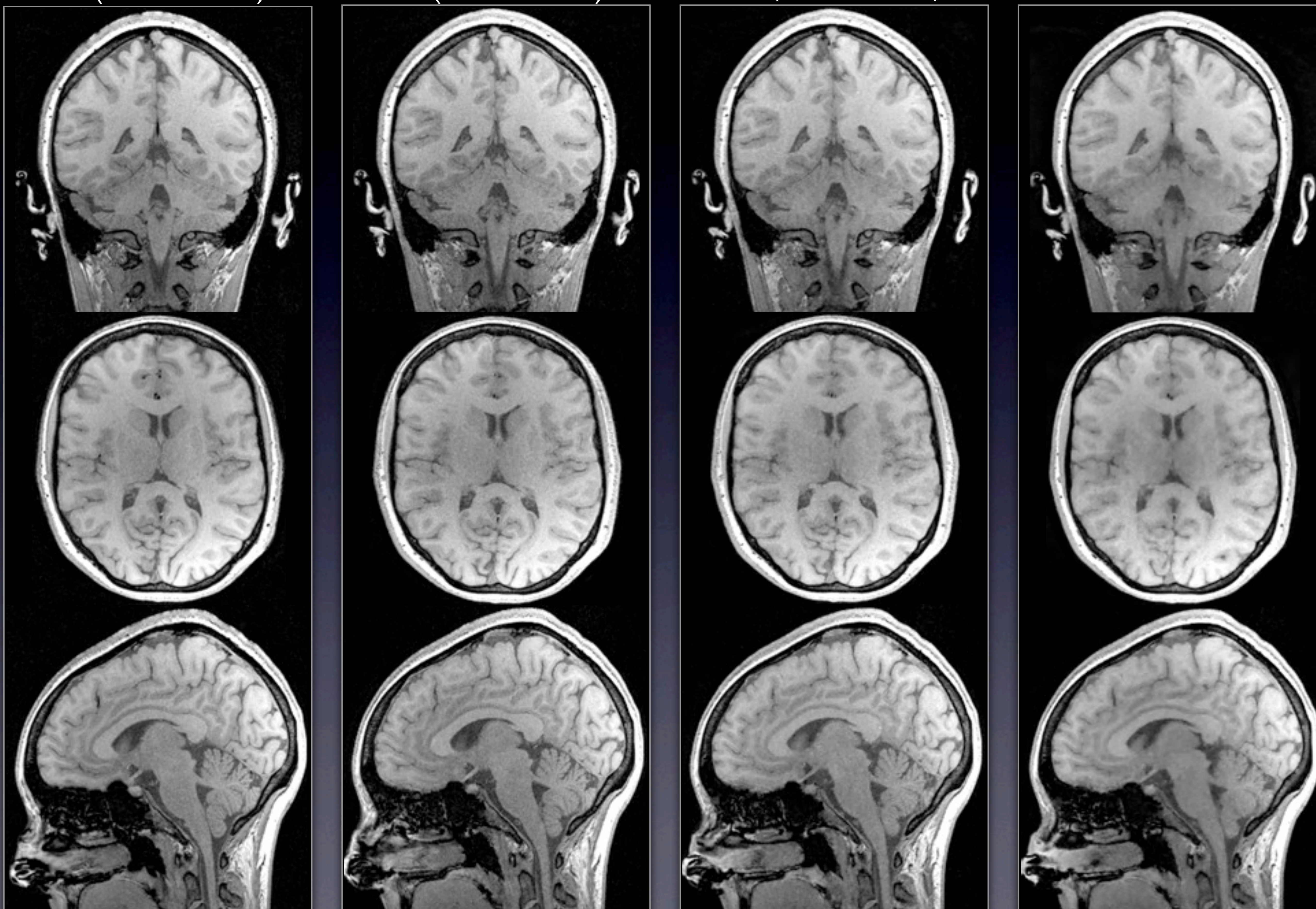


Phantom images

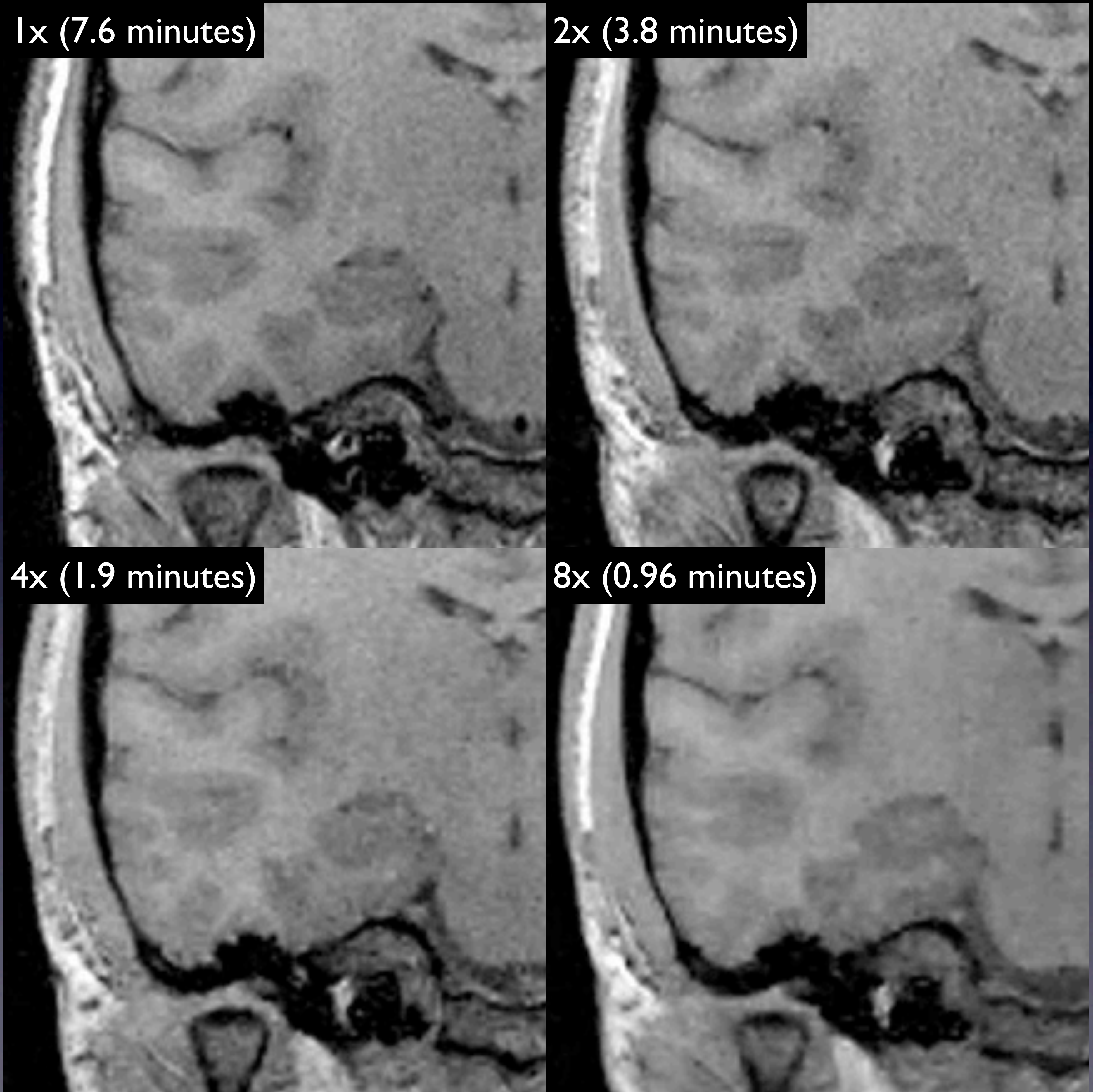


3D T_1-w SPGR: 256 x 256 x 256 (0.78 x 0.78 x 0.78 mm^3)

1x (7.6 minutes) | 2x (3.8 minutes) | 4x (1.9 minutes) | 8x (0.96 minutes)



3D T_1-w SPGR: 256 x 256 x 256 (0.78 x 0.78 x 0.78 mm^3)



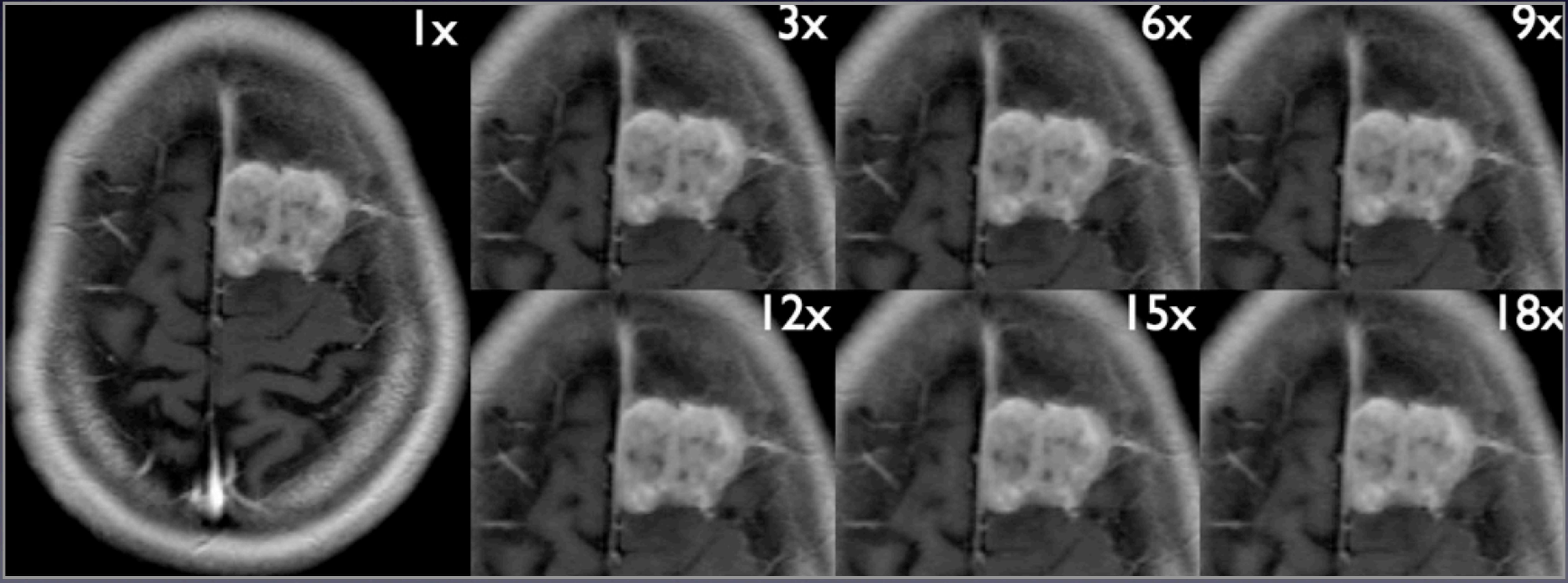
Dynamic Contrast Enhanced MRI

Retrospective undersampling

3D T_1-w SPGR: 256 x 186 x 10 (0.93 x 0.93 x 3.0 mm^3)

35 time frames, 10 s temporal resolution

l_1-SPIRiT with 4D wavelet, 3D finite differences, dynamic constraints



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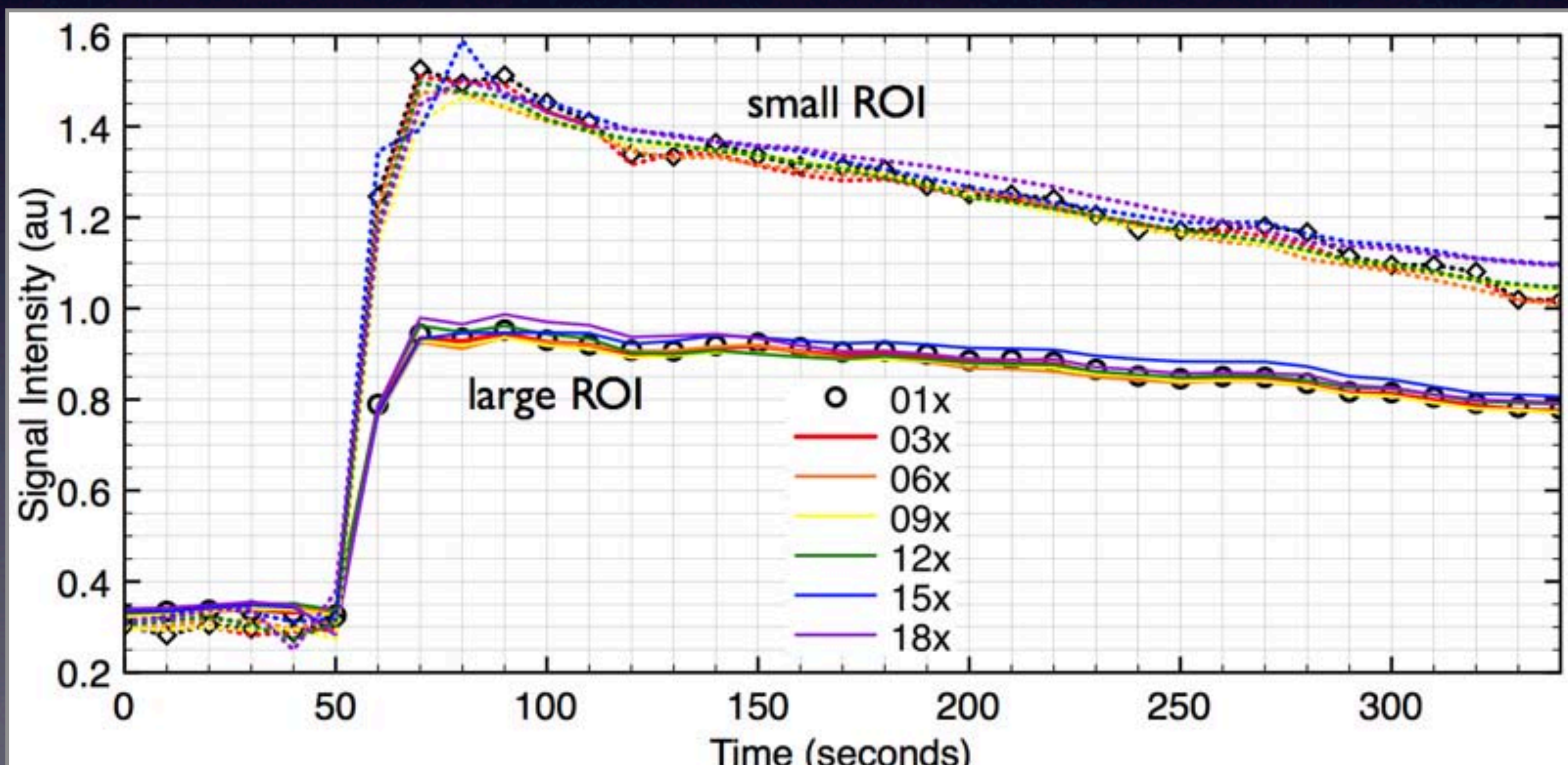
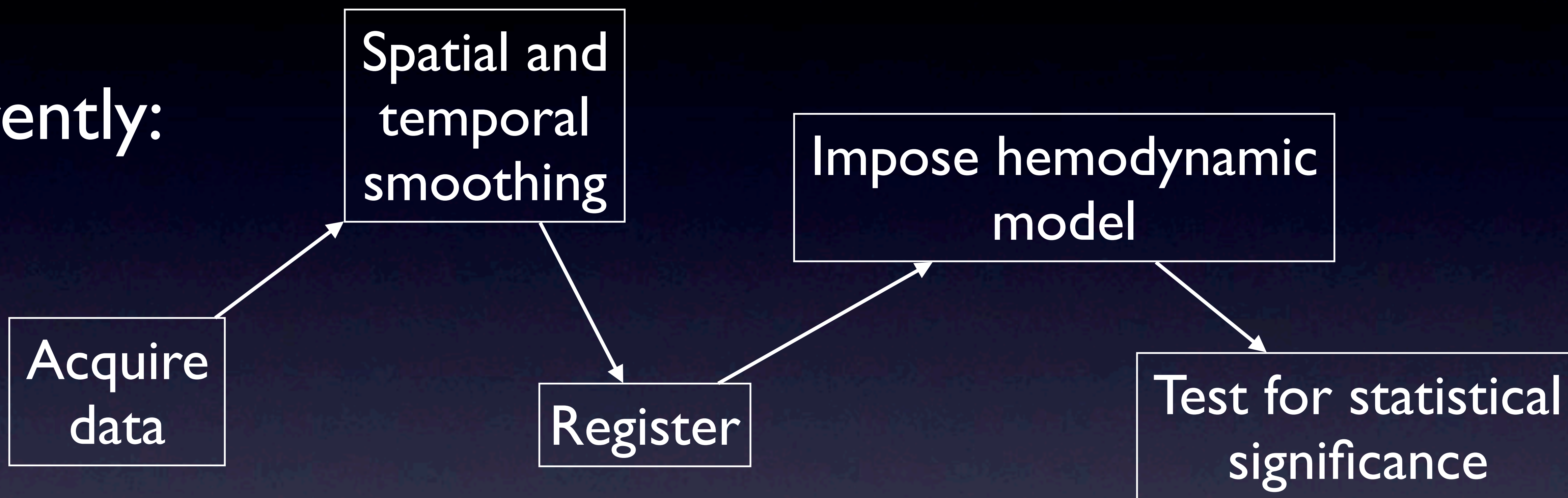


General Thoughts

- We’re going to see a lot more compressed sensing/constrained reconstruction in the future
- Vendors are watching the field very carefully
- Currently challenging to implement in a robust and reliable manner
- Collaborations are needed

General Thoughts (fMRI)

Currently:



Potentially:

<table>
<tr><td rowspan="2">Acquire data (appropriate sampling pattern)</td><td colspan="2">Reconstruct parametric maps</td></tr>
<tr><td>Regularize for:
spatial smoothness
temporal drift</td><td>Impose hemodynamic response</td></tr>
</table>

Questions?